November 2016 subject reports

Mathematical Studies SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–16	17–30	31–43	44–55	56–68	69–80	81–100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–4	5–6	7–8	9–11	12–14	15–16	17–20

The range and suitability of the work submitted

This was the first time that Mathematical Studies SL projects were uploaded and marked onscreen, with dynamic sampling being used. As usual nearly all of the candidates opted for a statistical analysis project. The topics were the typical ones, e.g. GDP and life expectancy, hours of sleep and grade average and sports. In between there were one or two non-statistical projects; Pythagorean triples and equilateral triangles drawn on their sides, modelling a dam for collecting water for the school and modelling a 105 cm jump. However, there were some projects that seemed to be more an essay on sociology than a Mathematical Studies project, showing that teachers did not give sufficient guidance to their candidates. Most candidates used surveys or Internet referenced sources to collect their data. It was pleasing to see sources referenced. The vast majority of projects had structure and developed logically. Most had at least some appropriate notation and terminology. More candidates this session were defining their variables. The conclusions drawn were mostly consistent with the results. Validity was, as always, the criterion least well addressed.

There are still a significant number of projects that only develop one process, either simple or further, repeating it a number of times, and considering it as separate processes. Also, some develop only further processes not realizing how this affects their project.

Candidate performance against each criterion

A: Most candidates were able to achieve level 2 as projects contained a statement of task, a title and a plan, if at times brief, for carrying out the task. Candidates mentioned the mathematical processes they would use but still, at times, there were processes not mentioned in the plan that were carried out in the analysis. This deprives the candidates of achieving more than level 2 in this criterion. Only a few candidates mentioned processes in the plan that they did not carry out. Most of the projects had a title.

B: Some candidates did not show the raw data collected so it was not possible to verify the tables or calculations. Apart from this, candidates were able to achieve level 2 since the data collected was sufficient and organized/reorganized ready for analysis. Level 3 was often not achieved because the candidates did not describe fully the sampling technique. Many did not discuss why and how they had chosen to sample, for example, 30 people. Also, there were some that included an Internet site to direct the reader to the raw data. Teachers should be aware that these situations should be discouraged. There were candidates that used a formula to calculate the number of observations from the population that must be included in the sample; however, they often did not explain where the formula came from nor what the parameters meant. Candidates should show that they understand what they are using.

C: Candidates generally used at least two simple mathematical processes along with a further process, either a χ^2 test or Pearson's product–moment correlation coefficient. There were times when the simple processes were not relevant to the task and this limited the award to level 2. More candidates than last session were aware of the need to apply Yates's continuity correction to a 2 by 2 matrix. At times candidates showed too few calculations and made too few interpretations in the simple processes. For example, a bar chart with no comment, a calculator generated r-value for the correlation coefficient and a brief comment regarding positive/negative correlation. Some candidates lost marks because of the lack of raw data, which meant that their calculations of simple and further processes could not be checked for accuracy. Also, some carried out one or maybe two simple processes, multiple times, which does not count for multiple processes. Occasionally a project contained a χ^2 test as the only process. If this is carried out correctly, showing all the steps, then it can be awarded 1 mark. Otherwise it scores 0 for this criterion.

D: All candidates drew at least one conclusion from their results. At times, there were inconsistencies which detracted from the work and led to level 1 being achieved. However, the stronger candidates usually had a detailed discussion of the results found and this was pleasing to see. Some projects had quite a few interpretations based on opinion and not the facts of their processes. These cases did not score more than 1 mark.

E: It was usually the stronger candidates who commented meaningfully upon the processes used and the results found. Some went on to discuss the limitations of their results. No attempt to fulfil this criterion was made by some candidates and this criterion was the least well addressed.

F: Most of the projects had some structure and developed logically. A few projects lacked explanation at each stage. In some projects the axes, especially on the bar graphs and line graphs were not labelled. Bibliographies/referenced sources were often seen in an appendix. Level 3 was not achieved mainly because, although the project was quite good, it was too simple. Another reason was a lack of explanation on how the categories for the χ^2 test were subdivided.

G: Appropriate mathematical language and notation was used in many of the projects. The main reason for not scoring well in this criterion was poor notation, such as using X for the symbol for χ. Often the variables were not explicitly described.

Recommendations for the teaching of future candidates

- Make sure that candidates include ALL raw data collected, either in the body of the project or the appendix.
- Ensure the simple processes used are meaningful and relevant to the task.
- Define the variables.
- Show some/all calculations that lead to the result.
- Ensure that the equation of the regression line, if calculated, is used.
- Make sure that all candidates read the assessment criteria and are fully aware of what they demand.
- Teachers should explicitly provide evidence, on the IA projects (preferably by annotating the pages directly) for awarding the different levels of achievement for the criteria.
- Give candidates clear guidance on how to complete a project and what should be explicitly stated at each stage.
- Give candidates examples that show good work, not so good work and bad work, so they can better understand the difference between them.
- Make sure that they understand the difference between the "statement of the task" and the "plan" and that a well developed plan, should include justification of the different techniques used.

Recommendations for IB procedures, instructions and forms.

It would greatly help the moderation process, if schools wrote comments related to each criterion, where the evidence is located on the body of the projects; some schools did and this was extremely helpful for the moderator. Schools should follow the upload instructions (available on IBIS) regarding annotating directly onto word-processed work. Examiners will see a static image of the work and cannot expand comments or move them to read text beneath them. To that end, comments should be fully expanded, but positioned in regions of whitespace (such as the margin) when the work is uploaded.

Teachers could, as an alternative, write their comments in the text box at the point of upload, stating the page where the evidence for awarding each level of achievement for the different criteria is located.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–13	14–27	28–40	41–52	53–64	65–76	77–90

The areas of the programme and examination which appeared difficult for the candidates

- Rounding off with specified accuracy
- Accurately reading from the cumulative frequency curve
- Interpreting set notations; subsets
- Exclusive disjunction
- Finding the range of a quadratic function
- Finding the angle of depression
- Using the derivative to find a point if the gradient is given
- Axis of symmetry of parabola and its equation
- Compound probability

The areas of the programme and examination in which candidates appeared well prepared

- Drawing a boxplot
- Shading a specific region in a Venn diagram
- Simple finance questions
- χ^2 test
- Solving simultaneous equations with a graphic display calculator
- Arithmetic and Geometric progressions
- Basic probability
- Finding a derivative
- Basic trigonometry questions

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

A few candidates were not able to respond adequately to the request to “write down the full calculator display” of the result. A significant portion of the candidates did part (a) correctly, but some candidates had their graphic display calculator set in radians. Many candidates gained follow through marks in part (b). Rounding to 2 decimal places was usually well done, but many had problems rounding to 3 significant figures. Generally there was a good performance with writing the answer in standard form in part (c).

Question 2

This question was usually well answered. Most candidates were able to read and interpret the cumulative frequency graph and answer well in part (a). Part b) was also well done. Many candidates who did not get full marks in part (a) received follow though marks in part (b).

Question 3

Very few candidates gained full marks in this question. It appeared that the symbol $\subset$ was not well understood. Many incorrect answers were seen in part (a). Part (b) was well attempted and answered.

Question 4

Part (a) was well answered, although many candidates did not give the final answer correct to 2 decimal place and lost the last mark. Overall part (b) appeared to be difficult for many candidates, although method marks were gained by most candidates.

Question 5

The truth table was generally well done. Many candidates omitted “if … then” or “but not both” in part (a). Very few candidates were able to understand the question in part (c) and answer it correctly.

Question 6

Many candidates answered this questions very well. Some candidates were not able to answer part (a). Part (b) was well done by most candidates. In part (c), some were confused about the values they needed to compare in order to draw a conclusion, and some were not able to provide justification for their conclusion.

Question 7

Part (a) was well attempted, but correct answers for part (b) were not often seen. Many candidates had problems finding the 40% increase of their answer to part (a). Some candidates lost marks for not including the units in their answers.

Question 8

Parts (a) and (b) were well done by most candidates. However, not many could solve the equations correctly by either using a graphic display calculator or algebraic methods. Many candidates incorrectly substituted into the formula for the percentage error and quite a few left the final answer as a negative number.

Question 9

This question was well answered by very few candidates. Many candidates had problems with correctly identifying the range of the function and had difficulties using correct notation to write it down.

Question 10

Most candidates managed to find the ratio, distance and total depth. A few used arithmetic progression formulae or calculated incorrectly the common ratio for the geometric progression.

Question 11

Part (a) was answered well for the most part. However, many found part (b) to be quite difficult, especially calculating $\frac{4}{3}\times$ part (a). Some candidates made use of the sine rule. Very few candidates calculated correctly the angle of depression.

Question 12

Many candidates placed correctly "p" but not "$1-p$" on the tree diagram given in part (a). Some candidates also used incorrectly $\frac{p}{4}$ and $\frac{(1-p)}{4}$. The fact that the probability was given as "p" and not as a specific number seemed to be a major obstacle for the candidates. There were not many correct answers in part (b).

Question 13

This question was well attempted. Most candidates were able to answer part (a) correctly. Many candidates were able to write the inequality in part (b), but were either not able to solve it or not able to correctly round their final answer.

Question 14

Most candidates found the derivative correctly in part (a). Some were able to correctly set the derivative equal to -10, but then could not solve the equation. A common error was to substitute -10 into the formula for x.

Question 15

This question was relatively well attempted. Some candidates correctly substituted 0 into the function, but did not evaluate $(1.1)^0$ as 1 and as a result lost the last mark in part (a). A common incorrect answer to part (a) was 12870 due to the candidates evaluating $(1.1)^0$ as 0. Most of the candidates scored at least one method mark in parts (b) and (c).

Recommendations and guidance for the teaching of future candidates

Check answers carefully: Candidates should be reminded to check their answers to ensure they are reasonable within the context of the question.

Pay attention to the required accuracy for the specific answers: Candidates should be reminded to give their answers to the accuracy required by the question, or to three significant figures otherwise. They should be taught that marks may be lost if the correct accuracy or the specified units are not used.

Know the command terms: Candidates should know all of the command terms so that they know what action is required in each response. They should also be careful in investing the appropriate efforts in the given task.

Learn to write succinct, clear, and well-grounded justifications: It is important that candidates learn to communicate clearly. Teachers should ensure candidates practise drawing conclusions and writing clear, succinct, and well-grounded justifications to support them.

Review past papers: Candidates should familiarize themselves with previous papers, their format, and key terms that are used.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–13	14–27	28–38	39–48	49–57	58–67	68–90

The areas of the programme and examination which appeared difficult for the candidates

- Drawing and using scales correctly, particularly where the scales are different for each axis.
- Correct and efficient use of the graphical display calculator where required.
- Reliability of using a regression equation to estimate a value.
- Understanding what is meant by the command ‘Show that’.
- In probability, understanding of the phrase ‘at most one’ and the concept of conditional probability.
- Finding the equation of a normal.
- Properties of a rhombus.
- Using a trigonometrical formula in an unfamiliar context.

- Conversion of metric units.
- The differentiation of negative powers.
- Realization of when an answer is unrealistic in the context of the question.
- The concept of 'least number'.

The areas of the programme and examination in which candidates appeared well prepared

- Using the graphic display calculator to find summary statistics.
- Venn diagrams.
- Calculating the midpoint, gradient and distance between two points.
- Finding the coordinates of the point of intersection of two lines.
- Drawing and suitably labelling the normal distribution curve.
- Finding the coordinates of the point of intersection of two lines.
- Using the graphic display calculator to find a probability for the normal distribution.
- Pythagoras' Theorem and correct use of the cosine rule.
- Correctly calculating areas of a triangle and a quadrilateral.
- Construction of the surface area of an open cylinder.
- Finding a root of a cubic equation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Scatter diagram and regression.

Part (a) proved to be more problematic than in previous years. The usual minority of candidates either did not label their axes properly or simply reversed their axes – consequently losing the first mark. More significantly however, were the number of candidates who seemed unable to read correctly from their horizontal axis and many of the given points were plotted as if the horizontal scale was 1 cm representing 5 hours rather than the given 2cm representing 5 hours. This proved costly to such candidates as not only did they lose most of the marks in part (a) but also the accuracy mark in part (c) for miss-plotting the calculated value of $\overline{x}$. Parts (b), (d) and (e) were testing the correct use of the graphic display calculator; the marks available here were for writing down values and candidates who showed a manual method for determining their values earned no additional marks for method and consumed a lot of time in showing these processes. For part (c) a significant minority failed to label the point as instructed using the letter 'M', and consequently lost a mark here. In part (f), many candidates seem to have been well drilled in the process of drawing the regression line and many straight lines were seen passing through their plotted point, M and intercepting the y-axis in the correct place. Part (g) was straightforward for those candidates who read the question properly but both marks were lost by candidates who simply substituted the value of 34 into their answer to part (e). Part (h), whilst only worth one mark, proved to be quite a discriminator; a surprising number of candidates were not able to correctly address the reliability of their estimate. Common errors included making reference to outliers, or stating the known points were not always very close to the regression line. It is clear the reliability of the regression equation is a concept which requires further reinforcement.

Question 2: Venn diagram and probability.

Drawing a correct Venn diagram from the information given proved to be accessible for the vast majority of candidates. Indeed, the only errors that seemed to be evident were either a missing rectangle around the circles or substituting '3' for x on the diagram. Part (b) proved to be somewhat of a discriminator with many incorrect methods seen. The most popular erroneous method was simply to write down a numerical equation, not involving x, but simply stating eight numerical terms, (with '3' conveniently substituted), equal to 66. This earned no marks at all. Some candidates performed slightly better by starting off with a correct equation but contrived to arrive at an invalid conclusion (losing accuracy). In part (c), the notation $n(B \cap C)$ seemed not to be understood by a significant number of candidates with erroneous answers of 2 or 12 being prevalent. In the case of the former, it seems that such candidates identified the number '12' in $B \cap C \cap H'$ and the number '3' in $B \cap C \cap H$. This was then incorrectly determined as two elements in the required subset. Arriving at the popular, but incorrect, answer of 12 leads one to assume such candidates simply thought that $n(B \cap C) = n(B \cap C \cap H')$.

In part (d), the probability element of the question proved to be quite a discriminator with many candidates failing to get beyond two marks. In part (d)(i) a significant number of candidates, ignoring the fact that 'at most one trip' includes no trips at all, wrote down the fraction $\frac{34}{66}$. In part (d)(ii), a correct numerator of 3 was often seen with an incorrect denominator of 66. Clearly, such candidates have a poor understanding of conditional probability.

Question 3: Geometry – straight lines and two dimensional figures.

This question proved to be quite challenging. The majority of candidates seemed to know that they needed to substitute the given coordinates of $(1, 4)$ into the given equation of the line, L_1, and showed that the left hand side of the equation equated to zero. This earned some credit but many then lacked a conclusion, linking their work back to the question, such as ' …*therefore A lies on L_1…*'. Parts (b) and (c) proved to be popular with many candidates seemingly well drilled in the techniques of finding the coordinates of the midpoint of a line segment and the length of a line segment. Part (d) however proved to be the downfall of many and quite a discriminator. Many candidates simply copied their method from part (a) and substituted the coordinates $(5, 12)$ into the equation of L_2. Whilst this showed that $(5, 12)$ is on L_2, it did not show that L_2 is perpendicular to AC. Indeed, a significant number of candidates tried to show that L_1 and L_2 were perpendicular to each other. In both cases, such candidates earned no marks for this part of the question. Candidates who attempted to draw a diagram for this part of the question invariably found this to be helpful and successful candidates (with or without a diagram) correctly found the gradient of AC to be 2 and subsequently the gradient of the normal to be $-\frac{1}{2}$. The vast majority of candidates who reached this stage of the problem were able to complete to a satisfactory conclusion. Part (e) was reasonably well answered with at least one mark earned by many for simply attempting to solve the two given equations. In part (f) some candidates interpreted *five significant figures* to be equivalent to *five decimal places*. It was also surprising that a significant number of candidates found $\sqrt{\frac{45}{2}}$ (4.7434) instead of $\frac{\sqrt{45}}{2}$ (3.3541). Part (g) was deliberately chosen as a discriminator and was either very poorly answered or not attempted at all by the vast majority of candidates. Centres should be reminded that under prior learning topics, their candidates should be aware of *simple two dimensional shapes and their properties.* As the diagonals of a rhombus bisect at right angles, the simplest approach to the solution of this part of the question required the candidate to use their answer to part (c) and combine it with their answer to part (f). Again, candidates who drew a sketch of the rhombus invariably found this helpful in attempting this part of the question.

Question 4: Normal distribution

Whilst this question was generally well answered, some candidates failed to make a start suggesting the entire topic was alien to them. In response to part (a), many were able to draw a normal distribution diagram clearly identifying the mean and giving an indication of standard deviation. Making effective use of their graphic display calculators, many candidates found the correct probability in part (b)(i). Similarly, many candidates were able to find the expected daily recycling cost in part (c). Hoping to pick up at least some method marks if their probabilities were incorrect, some candidates attempted to support their answers with calculator notation. Centres should inform their candidates that this notation, in itself, earns no marks and centres should encourage their candidates to draw a diagram identifying the area under the curve where a probability is either needed to be found or is given and shade any relevant area. In this way, method marks can be earned if the probability or value required is incorrect. In part(d), a surprising number of candidates incorrectly found the top 3% and gave the incorrect answer of $353.385\ldots$

Question 5: Non-right angled trigonometry and areas.

Whilst it was rare to see a graphic display calculator used in radian mode, centres would be well advised to instruct **all** their candidates on how to ensure they have set their calculator to *degree* mode when attempting trigonometry questions. Parts (a) and (b) were tackled reasonably well with good use of Pythagoras' Theorem and the cosine rule. A minority of candidates mixed up their sides in part (b) but otherwise many scored well on these two parts. The majority of candidates used the correct formulae in parts (c) and (d) but lost a mark if the units were incorrect in either part. A minority of candidates used $\frac{1}{2}\times\text{base}\times\text{height}$ for triangle ABD but the majority of the remaining candidates confidently used the correct formula. Part (e) however proved to be quite a discriminator. Whilst candidates were able to show the basic processes of trigonometry and mensuration in the first four parts of the question, this part required candidates to set up an equation from the data given. Many did not know where to start with this and consequently left the part unanswered. Others simply assumed that triangle APB was a right angled triangle or attempted to use the sine rule on triangle APB. Only a small minority used the data given in the stem of the question and set up the necessary equation to find the length of AP. A correct (or incorrect) answer for part (e) enabled many candidates to correctly use the cosine rule again to find the length of the fence, BP in the final part of the question. Unfortunately, no answer in part (e) meant that no marks were available in part (f).

Question 6: Mensuration, functions and calculus

A significant number of candidates seemed to forget the base and simply wrote down $2\pi rh$ for part (a). Of those that did add on the base, some failed to achieve both marks as they 'simplified' $\pi r^2 + 2\pi rh$ to $3\pi r^2 h$ thus losing the second mark. Part (b) proved to be quite challenging as the vast majority of candidates did not seem to be able to convert the units correctly and, as a consequence, $50\ \mathrm{cm}^3$ proved to be a popular, but erroneous, answer. In part (c), candidates failed to appreciate that they needed to give an equation for the volume of **this** container and simply gave $V = \pi r^2 h$ (earning no marks). With many incorrect answers to part (b) coupled with some incorrect answers to part (a), very few completely correct answers were seen in part (d). Indeed, for those candidates who incorrectly converted in part (b), the appearance of $1\,000\,000$ in the given formula should have been an indicator that something was wrong if their final term only involved a factor of 100. At this stage of the question, candidates were able to recover by making a valid attempt at differentiating the given equation from part (d). Some candidates had difficulty with the negative index but, generally, attempts at calculus earned something in part (e). In part (f), many equated their derivative to 0 to find the value of *r* which minimizes A. The majority of candidates who attempted a solution did so by using their graphic display calculator to find the value of r. Few attempted an analytic approach for this part; those who did so were less successful than those who used technology. Unfortunately some made a transcription error transferring the number from their calculator to their script. Candidates should be aware that clerical errors are more likely when dealing with large numbers and so extra care should be taken. At this stage of the paper, if candidates had not run out of time, the final two parts were accessible and much correct working was seen with candidates earning at least three out of the possible five marks. The marks which tended to be lost were for either an unrealistic answer for the number of cans of water-resistant material or for a final answer which was not correctly rounded.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- develop the steps towards the given answer, in a 'Show that' question, rather than by starting from the answer which is given.
- use the graphic display calculator efficiently – there is no requirement to write down the steps followed on their graphic display calculators to reach their answers.
- show any working with unrounded figures and then give answers exact or to three significant figures. Remember, follow through answers are generally not awarded if working is not seen. Premature rounding can be an issue for multi part questions and candidates should show and use unrounded intermediate answers as much as possible.
- critically examine their answers to see whether or not they are sensible in the context of the problem set. If units are given in the information, always give the required units in answers.
- not cross out their work unless it is to be replaced – crossed out working earns no marks at all.
- give consideration to the weight of a question – one mark is approximately one minute of time for the paper. Lengthy explanations are not necessary when the question is only worth one or two marks.

Teachers should make use of the *Online Curriculum Centre* (OCC) http://occ.ibo.org/ibis/occ/guest/home.cfm where questions can be raised in the forum and teaching ideas and good practice may be shared.