November 2015 subject reports

Mathematical Studies SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-19	20-37	38-49	50-61	62-74	75-85	86-100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-8	9-11	12-14	15-16	17-20

The range and suitability of the work submitted

Nearly all of the candidates opted for a statistical analysis. The topics were the usual ones but there were one or two different tasks that were clearly of personal interest to the candidate. It was pleasing to see that many candidates/schools were aware of the new (as of 2014) criterion requiring two simple processes and then a further process. Projects were much more focused and plans were well presented on the whole. Most of the samples from the schools had the full range of marks. If marks were below 5 then it was usually because the project was incomplete. Generally, candidates used surveys or Internet referenced sources to collect their data. Some teachers ignored the fact that, if there are no simple processes in the project, then the first two further processes are counted as simple. The vast majority of projects had structure and developed logically. Most had at least some appropriate notation and terminology. Variables often were not defined and this detracted from the work. The conclusions drawn were mostly consistent with the results. Validity was the criterion least well addressed although saying this there has been an improvement here this session. Finally, not all teachers are checking, even randomly that calculations are correct. This leads sometimes to a significant difference between the teacher award and examiner award in criterion C.

Candidate performance against each criterion

A: Candidates generally were able to achieve level 2. All projects contained a statement of task and usually a plan, if at times brief, for carrying out the task. Often candidates mentioned the mathematical processes that they would use but did not justify the reason for choosing each of the processes carried out. Occasionally processes not mentioned in the plan were carried out in the analysis or processes mentioned in the plan were not carried out. This deprived the candidates of achieving more than level 2. To be awarded level 3 there should be no surprises when reading the project.

B: Most candidates were able to achieve level 2 since the data collected was sufficient and organized/reorganized ready for analysis. Level 3 was not often achieved, as the candidates did not describe fully the sampling technique. More work is needed on this criterion with regard to sampling. Only the very best projects included any details of the sampling technique selected. Some candidates needlessly threw away marks by failing to include their raw data.

C: It was good to see most of the candidates using at least two simple mathematical processes along with a further process, either a χ^2 test or a scatter diagram and Pearson's product–moment correlation coefficient. Saying this, at times the simple processes were not relevant to the task and this limited the award to level 2. The stronger candidates often used (after the simple processes) two further processes in their analysis and this was pleasing to see (although, of course, not a requirement). In some schools candidates knew that they needed to apply Yates's continuity correction to a 2 by 2 matrix. In other schools they did not. Many candidates had expected values less than 5 and this was not mentioned by the teacher. In this case they must regroup their data for the test to be valid. Candidates occasionally found the equation of the regression line then the correlation coefficient. This should be performed the other way around. Often the regression equation was quoted but not used. Overall, the candidates showed too few calculations in the simple processes. Calculator generated results appeared without working or interpretation and this made it difficult to assess understanding. Also, when the result was not correct it lowered the award to level 1.

D: Candidates were, on the whole, able to draw one conclusion from their results. The stronger candidates had quite detailed discussions of their results. A project reads well if partial interpretations are written after each mathematical process. Some candidates still give irrelevant or unsupported conclusions or write down their own personal beliefs.

E: It was usually the stronger candidates who commented meaningfully upon the processes used and the results found. Some went on to discuss the limitations of their results. No attempt to fulfil this criterion was made by other candidates and this criterion was the least well addressed.

F: Most projects had some structure and developed logically. However, a few projects did not contain comments throughout the task but just had an overall conclusion at the end and this detracted from communication. Some projects were rather short and did not reflect the time requirement. Bibliographies/referenced sources were often seen in an appendix. Level 3 was not achieved mainly because, although the project was quite good, it was too simple and did not reflect the time commitment. Photographs of work done on paper should be discouraged as the projects will have better presentation if the work is typed and graphing software used.

G: Many candidates only scored one mark for this criterion. Many candidates are not using the correct symbol for χ or for multiplication. At times variables were not explicitly described. Some candidates still refer to “finding a correlation” rather than a relationship with reference to the χ^2 test.

Recommendations for the teaching of future candidates

- Read the subject reports
- Include ALL raw data collected in an appendix and the sampled data, for example every fifth data point used for analysis, in the body of the project
- Explain sampling to the candidates
- Ensure the simple processes used are meaningful and relevant to the task
- Define the variables
- Show some/all calculations that lead up to the result
- Use Yates’ continuity correction if the degree of freedom is 1 in a χ^2 test
- Make sure that all the expected values in a χ^2 test are greater than 5
- Ensure the regression line is relevant before drawing it on a graph
- Ensure, when found, that the equation of the regression line is used
- Use equation editor to ensure that correct notation is used
- Encourage candidates to fully explain the reasons for using mathematical processes described in their plan
- Have candidates assess previous projects so that they understand the assessment criteria
- Encourage candidates to use a different range of topics
- Make the candidates aware of the fact that repeated processes count as a single process

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-17	18-35	36-46	47-57	58-68	69-79	80-90

The areas of the programme and examination which appeared difficult for the candidates

Few candidates were able to identify whether numbers were natural, an integer, rational and real. Only the best candidates correctly substituted into the percentage error formula including the use of the modulus signs. Although candidates were familiar with the χ^2 test, the conclusions often lacked detail with vague statements such as "it is rejected". Some candidates found modelling difficult; for example finding the population percentage change and using that rate to estimate the population in the future. Weaker candidates lacked problem solving skills, and so were unable optimize an area given the perimeter of the rectangle when it was asked in context. Many candidates were unable to solve linear or quadratic equations. Other specific weaknesses are detailed in the question analysis.

The areas of the programme and examination in which candidates appeared well prepared

Overall the candidates found this examination paper accessible, with many attempting the later questions. Candidates found the descriptive statistics and tree diagram questions straight forward. Almost all candidates were able to attempt the logic, arithmetic sequences, linear functions and exponential functions questions. Most candidates could correctly convert currencies and were able to substitute values into a compound interest formula or enter the values into the *solver* on their calculator. Other specific strengths are detailed in the question analysis.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Very few candidates managed to answer this question without any errors. The candidates seem not to understand the number system, in particular that rational numbers are a subset of real numbers. Candidates did not recognize that the set of the real numbers, contains all the numbers that are studied in this course. Furthermore many candidates did not know that all integers are real.

Question 2

This question was very well attempted. Most candidates found the median correctly. Some did not rearrange the numbers first. Some confused mean, median and mode. The box-and-whisker graph was mostly correct but not all candidates used a ruler.

Question 3

Candidates were able to convert currencies but some made mistakes in finding the percentage error dividing by the approximate value rather than the exact value or converted the amount spent rather than the amount remaining. It is important candidates read the question carefully and follow the stated level of accuracy.

Question 4

Most candidates could find the common difference of an arithmetic sequence but could not find the sum of the series.

Question 5

The weaker candidates sampled with replacement in the tree diagram so had not read the stem of the question. Compound probability was only calculated correctly by the stronger candidates.

Question 6

Given a table of observed values almost all candidates correctly found the grand total and the calculated χ^2 statistic. Many candidates used the correct terminology in stating the null hypothesis. The weakest aspect was providing a correct reason for the conclusion of the test. There are still candidates that do not fully understand what is expected when asked to justify this type of question. Occasionally candidates sketched the χ^2 distribution to support their conclusion.

Question 7

Not all candidates were able to convert from metres to kilometres. Some of the weaker candidates labelled the right angle as the angle of elevation. Stronger candidates scored full marks and so could use right angle trigonometry to find a missing angle and the theorem of Pythagoras to find the hypotenuse.

Question 8

Many candidates did not calculate the population percentage change correctly but most picked up follow through marks for putting their answer in standard form. Some wrongly used arithmetic rather than geometric formulae (or could have used an exponential function) to estimate the future population.

Question 9

Almost all candidates attempted to fill in the truth table but only stronger ones did so correctly. It was disappointing to see candidates lose marks for the omission of "if" and "then" in their conditional statements. Candidates found it easier to write, in words, a compound statement from the logical symbols rather than vice versa. There was some confusion in determining whether it was a tautology, logical contradiction or neither.

Question 10

Many candidates could draw the straight line (but not necessarily accurately) and substitute a value into a linear equation to solve for the unknown. Only the very strongest candidates could find the temperature at which Celsius and Fahrenheit have the same numerical value; many did not attempt this part.

Question 11

Most candidates were able to find the future value using their calculator *solver* or the compound interest formula. Depreciation was not universally understood with some using the simple interest formula, which is not in the current curriculum.

Question 12

Candidates found the common ratio of a geometric sequence straight forward though the reciprocal of this ratio was a commonly seen wrong answer. Only the better candidates could correctly use the geometric sum formula correctly.

Question 13

Many candidates knew the area of a rectangle is the product of its length and width but they were unable to use the perimeter to find the area in terms of x. Using calculus to find the maximum area of the garden was only done correctly by the very strongest candidates.

Question 14

Many candidates found this to be a challenging question. The axis of symmetry of the quadratic function was often stated as $x = -b / 2a$ (as given in the formula booklet). Very few managed to find the correct values for a and b although a few picked up a method mark for putting $(6, 0)$ or $(-4, 0)$ into the equation. Likewise, there were very few correct answers for the y-coordinate of the vertex. Many candidates left this question unanswered.

Question 15

It was pleasing to see many candidates achieve full marks on this question but their “working”, if seen, suggested that the exponential function was solved by trial and error. Some did not follow the instructions to round to the nearest whole number.

Recommendations and guidance for the teaching of future candidates

Read the syllabus to make sure everything is covered and use the markscheme and examiner report to ensure a thorough understanding of the examination specifications including command terms. For example "draw" requires the use of ruler for a graph. Tell the candidates to be conscious of time.

“I used my calculator” will not gain method marks. In paper 1 full working should be shown to allow the opportunity for partial credit if answers are incorrect. Unrounded answers should be written down. Candidates should label their work in the working box, linking it to the correct question part. Candidates should be taught to make sure the calculator is in degree mode.

Candidates should read the question carefully, especially any words in bold, and practise with past IB papers to ensure candidates are familiar with the notation and wording of questions. Candidates should use the formula booklet throughout the 2 years of the course so that they know what is in it and when to use it. Time management seems to be a recurrent problem. Candidates should practise under examination conditions and be aware that they are expected to be able to answer at a rate of approximately one mark per minute.

Pencil should only be used for diagrams and graphs; all working and final answers should be written in pen as the pages are to be scanned for the examiner.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-17	18-35	36-46	47-56	57-66	67-76	77-90

General comments

The areas of the programme and examination which appeared difficult for the candidates

- Determining whether two events are independent
- Calculating conditional probability
- Determining the reliability of an estimate from a regression line
- Producing relevant sketches for Normal distribution problems
- Providing a suitable mathematical reason to determine whether a function is increasing or decreasing
- Finding the equation of a tangent
- Using the graphic display calculator to find an x-intercept or a point of intersection
- Calculating the curved surface area of a cone

The areas of the programme and examination in which candidates appeared well prepared

- Producing a Venn diagram from given data
- Calculating simple probability
- Accurately plotting paired data
- Calculating the Pearson's product–moment correlation coefficient and correctly interpreting the meaning of this value
- Finding and using the equation of the regression line
- Trigonometry of a non-right angled triangle
- Finding the derivative function with an unknown parameter
- Gradient of the normal from the numerical value of the gradient of the tangent
- Volumes of a cylinder and a hemisphere

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Venn Diagram and Probability

The vast majority of candidates were able to draw a labelled Venn diagram and correctly place the given information in the appropriate subset. It was rare to see '2' rather than 'x' placed in the intersection of all three sets. Such candidates lost one mark in part (a). The other mark which was sometimes lost in part (a) came as a result of the failure by some candidates to draw a box around their three sets. The vast majority of candidates handled the first two parts of part (c) correctly but very few candidates were able to give the required reasoning in part (c)(iii) as to why the two events were not independent. A significant number of candidates simply confused this concept with mutually exclusive events. Part (d) proved to be quite a discriminator with many candidates failing to recognize that the problem was one of conditional probability and instead calculated probabilities for a compound event.

Question 2: Regression

The majority of candidates scored well on this question and many correct scatter diagrams were drawn for part (a) with labelled axes and the correct plotting of points. In part (b), many candidates were able to correctly use their graphic display calculator with the required answer for the product–moment correlation coefficient seen on the majority of scripts. It was indeed rare to see a script where the candidate had written down the coefficient of determination rather the product–moment correlation coefficient. The words *strong* and *positive* were required for part (b)(ii) and these words were seen in abundance. Except for the occasional rounding error for the mean score of the examination marks, many correct answers were seen for part (c). Part (d) proved to be somewhat of a discriminator as the plotting of their point M was sometimes out of tolerance and, as a consequence, both marks were lost here. Much correct working was seen in parts (e) and (f) with many correct equations for the regression line seen followed by correctly substituting the value of 8 into this equation and many correct answers of 42.3 were seen for part (f). It was clear that candidates did follow the requirements of the question here as the vast majority used their equation rather than the graph of their regression line to find the required answer. Candidates seemed well drilled in the process of drawing a regression line with the majority of candidates showing a straight line passing through M and intercepting the y-axis at their 25.1. The final part of the question, part (h), proved to be quite a discriminator with a surprising number of candidates not able to correctly address the reliability of their estimate. Common errors included comparing 89 with a value obtained from their equation, or by making reference to the value of r found in part (b).

Question 3: Normal Distribution

Many correctly drawn and labelled diagrams were seen in part (a). In the remaining parts of the question however, there were few attempts to show any method by either drawing a diagram or writing down a probability statement. This did not cause a problem if the candidate wrote down the required answer but it then became an all or nothing mark. In part (b), the majority of candidates determined correctly that a weight of 94 kg is two standard deviations below the mean. A significant number of these candidates however wrote the answer of 0.975 for part (ii) with no justification and earned no marks here. A labelled diagram showing the correct region indicated or a statement such as $\mathrm{P}(\mathrm{weight} > 94)$ would have at least earned these candidates a method mark. Part (c) was generally done well, but again some candidates lost both marks in part (c)(i) by simply writing down an incorrect probability with no method (labelled diagram or probability statement) shown at all. Part (d) proved to follow a similar pattern with candidates either scoring 2 marks or none at all. As in previous examination sessions, candidates appeared more adept at finding the probability of an event, but continue to find more difficulty with the area under the curve as was evident in responses to part (e).

Question 4: Non-right angled Trigonometry

A significant number of candidates lost marks on this question as a consequence of assuming that triangle ABC was a right-angled triangle. Such candidates scored no marks in part (a) but were able to pick up marks later on. Many more candidates however correctly used the cosine rule resulting in many correct answers of 49.3° seen. Indeed, it was rare to see a radian answer here although on at least one script, a candidate was seen to be working in gradians. Candidates seemed to be well prepared to avoid a units penalty as many scripts showed correct units for part (b). Part (c) was quite straightforward with the vast majority of candidates showing that angle $\mathrm{BDC} = 60^{\circ}$ and many candidates correctly used the sine rule for the last two parts of the question. As part (e) was a *show that* question, many candidates showed good work in their approach to the solution but lost the final mark because a valid conclusion was not stated.

Question 5: Functions and Calculus

Part (a) was quite a discriminator with the majority of candidates having difficulty using appropriate mathematical terminology to explain whether the function is increasing (or decreasing) at $x = -3$. Candidates fared better in parts (b), (c) and (d) and even the introduction of the parameter *n* caused few problems with the requirement to differentiate. The correct coordinates of the local minimum point were found by the majority of candidates although it seems that a significant minority misread the scales on the graph and $(4, -625)$ was a common, but erroneous, answer. Although a significant number of candidates started with $n = 10$ and then continued to show that $n = 10$ using a circular argument, many candidates either correctly started with $f'(4) = 0$ or correctly substituted their answer for part (d) into $f(x)$. Despite some arithmetical errors, most who started with a correct statement arrived at the required solution. In part (f), it seemed that not all candidates realized that $f'(-1)$ was the gradient of the tangent line at $x = -1$ and, as a consequence, there were many varied, but erroneous attempts to find the equation of the tangent in part (ii). These attempts, plus some poor arithmetic in part (i), led a substantial minority of candidates to achieve only ***(A1)*****(ft)** for part (iii) here. Overall, parts (g) and (h) were not well attempted. Even those who understood the demands lost marks due to accuracy or expressing their answer as coordinate pairs.

Question 6: Mensuration

Much correct working was seen in part (a) as the majority of candidates used the correct formula for the volume of a cylinder. The correct formula for the cone was used by many in part (b) although poor arithmetic led many candidates to an incorrect value of h. In part (c), many determined the radius of the hemisphere correctly from their h, but surprisingly, a significant number then went on to substitute $r = 4$ to find the volume of the hemisphere. Part (d) required candidates to carry out two processes: (*i*) to find the slant height of the cone using Pythagoras followed by (*ii*) the calculation of the curved surface area of the cone. For some candidates, step (*i*) was ignored and a substitution of their $l = 5.07$ into the formula for the curved surface area of the cone led to many candidates scoring no more than one mark here. Indeed, that mark was only achieved if the candidate had correctly rounded their decimal answer to the nearest cm^2.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- show all working (to at least four significant figures) and give answers to at least three significant figures. Remember, follow through answers are generally not awarded if working is not seen. Premature rounding can be an issue for multi part questions and candidates should show and use unrounded answers as much as possible.
- critically examine their answers to see whether or not they are sensible in the context of the problem set. If units are given in the question, always give the required units to answers found.
- not cross out their work unless it is to be replaced – crossed out working earns no marks at all.
- give consideration to the weight of a question. Lengthy explanations are not necessary when the question is only worth one or two marks.
- be conversant with appropriate terminology for each area of the course. This is particularly important when being asked to explain their reasoning.
- ensure that they are fully conversant with the formulae which appear in the information booklet, including exactly where these formulae are to be found in the booklet, prior to the examination.