MATHEMATICAL STUDIES SL

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 17	18 – 33	34 – 45	46 – 58	59 – 70	71 – 82	83 – 100

Standard level Project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

The range and suitability of the work submitted

This is the last set of projects marked under the “old” criteria.

Once again the projects were mainly statistical in nature with candidates attempting to verify a stated hypothesis. Many candidates produced interesting projects but many others appeared to be doing the bare minimum of work to achieve reasonable levels of achievement. Almost all the tasks chosen were appropriate for a Mathematical Studies SL project. In a few cases, the topic chosen was not an appropriate one and this was reflected in the analysis part where no or very few mathematical processes could be applied.

Many projects involved questionnaires or surveys but a copy of the questionnaire or survey was not always included with the project. Some candidates did not include their raw data, which precluded cross-referencing of data and checking of mathematical processes.

In some cases, where the candidate collected their own data, the data collection process was not given in sufficient detail to allow for the quality of the data to be assessed.

A number of candidates omitted all simple mathematical processes. In this case the first sophisticated process is considered “simple”. A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering their test invalid. Candidates also incorrectly drew conclusions about "correlation" based on their chi-squared test of independence. Few teachers picked up on these mistakes.

Many more candidates are relying on technology to do the mathematics for them and often do not do any mathematics themselves. Any mathematical processes which only use technology are considered simple.

More and more candidates are producing very short projects which do not reflect the 20 hours allocated for school work plus approximately the same amount of time outside of the classroom.

A few schools seem to be encouraging candidates to follow a set format. Whilst this does focus the candidate the overall result is, unfortunately, a lack of originality to the task.

The range of mathematics that was once seen is now significantly diminished. However, there were some candidates who produced wonderful projects that achieved high levels in almost every assessment criterion.

It was evident that the guidance provided by the teacher varied from school to school.

The comments made by the teachers on the 5/PJCS forms were often clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against each criterion

Criterion A

Most candidates produced a statement of task and described a plan that they would follow but not all of them explained the steps they would be taking or the mathematical processes that would allow them to achieve their goal. It is important to follow the stated plan. If the plan is well documented, then the project usually flows from there.

Criterion B

Many candidates collected sufficient data so as to facilitate their analysis. However, for other candidates the data was limited in quantity or they did not describe their sample selection and data collection process in sufficient detail to allow for the assessment of the quality of the data. Not all candidates set up their data in tables ready for analysis. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the tables of data or the mathematical processes used. The candidates should be aware that having a lot of data does not always mean that the work has the quality needed to gain full marks in this section. If data is too simple then it limits the mathematical analysis that the candidate can perform as well as the quality of the discussion on their results. When secondary information is used, candidates must clearly identify the source.

Criterion C

Fewer candidates this session omitted simple mathematical processes. A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering the test invalid. Many candidates were using a general statement of task as their null hypothesis instead of writing clear null and alternative hypotheses.

Some candidates confused the coefficient of determination with the correlation coefficient suggesting that the difference between the two is not clear to them.

Some candidates included only simple mathematics because their projects did not lend themselves to sophisticated techniques. Many used technology only to perform sophisticated techniques without realizing that this is considered as simple mathematics since the candidate has done no mathematics by themselves. Also, it does not show the moderator that the candidate has understood the process. Some candidates introduced mathematical processes that were totally irrelevant. This can actually result in the candidate losing marks. Teachers must check the accuracy of candidates' mathematical processes before awarding an achievement level in Criterion C.

Criterion D

Most candidates produced at least one result that was consistent with their analysis but few candidates produced thorough explanations of what had been found, calculated and observed. Some candidates did not write partial conclusions after each process but wrote an overall conclusion at the end. This hinders the flow of the project. Some candidates attempted to justify their results based on their own personal beliefs rather than the mathematics that they had performed.

Criterion E

More candidates are now commenting successfully on validity.

Their discussions often centre on data collection but more candidates are now attempting to comment on the validity of the processes themselves. Some candidates are beginning to add sensible suggestions for extensions of their project.

Criterion F

Most of the projects were structured and communicated well. It is also important to ensure that the notation and terminology is correct otherwise this restricts the mark that can be awarded for this criterion. Many candidates lost marks due to errors in either notation or terminology.

Criterion G

The majority of teachers appear to have awarded marks appropriately. However, some schools abuse this criterion by giving full marks to all candidates when the quality of work was very simplistic and hastily done.

Recommendations and guidance for the teaching of future candidates

Teachers can help candidates in many ways:

- Give them examples of "good" projects so that they know what is expected of them.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind candidates that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Check that the mathematics used in the project is relevant and correct.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are greater than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.

- Encourage candidates to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, and computer notation.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them.
- Inform candidates about sampling techniques.
- Remind them to include all raw data either in an appendix or as part of the task.
- Remind them of the importance of including simple mathematical processes in their projects.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals and set interim deadlines to monitor the progress of the project and catch any major deficiencies in good time.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 28	29 – 41	42 – 53	54 – 64	65 – 76	77 – 90

The areas of the programme and examination which appeared difficult for the candidates

The areas of the course which proved difficult for some candidates included statistics; especially finding the standard deviation, logic; filling in the truth table and finding the converse proposition, geometry; using DCB = $\tan^{-1}(0.6)$ and finding the area of a non-right angle triangle, arithmetic sequences, curve sketching, geometric sequences; set in context, and financial mathematics; using the table to calculate interest.

The areas of the programme and examination in which candidates appeared well prepared

The questions involving Venn diagrams, cumulative frequency graphs, tree diagrams, χ^2 tests, differential calculus, graphs of trigonometric functions and exponential models were confidently answered, with relatively few candidates leaving these questions blank.

Most candidates appeared to be familiar with the functions of their graphic display calculator.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Statistics

Most candidates could state whether the data was discrete or continuous and find the mode however the calculations to find the mean, median and standard deviation appeared problematic for some candidates. A significant number of candidates gave the mode as 37 rather than 0. Many did not appear to use their graph and some obtained the incorrect value of 1.47 from their graphic display calculator.

Question 2: Number sets and Venn diagrams

This question was done well by most candidates. The most frequent error was to omit the placement of 1 and 5 or to include 0 in the set of even integers.

Question 3: Logic

Most candidates were able to write the compound proposition in words, however many were not able to write the converse in symbolic form. While they were able to fill in the third column of the truth table, many were unable to complete the fourth column correctly.

Question 4: Obtaining information from the graph of a cumulative frequency distribution

Many candidates gained full marks on this question although a significant number could not find the interquartile range.

Question 5: Geometry and trigonometry

This question was not well answered. Many candidates could not use Pythagoras' theorem correctly and many failed to appreciate the significance of tan(< DCB)=0.6 and calculated the size of the angle DCB, rounding it to 31°. Unfortunately, this method led to an inaccurate value for DB. Finding the area of triangle ADC was also difficult for many who did not realize that they needed to do a subtraction of triangle areas. Candidates who tried to find side lengths and angles for triangle ADC were generally unsuccessful in calculating its area.

Question 6: Arithmetic sequence

Many candidates gave an answer of 8 rather than -8 but were awarded follow through marks in parts (b) and (c) where working was shown. Some candidates appeared unaware that the common difference in both the AP formula for a term and for a sum is multiplied rather than added or subtracted. Candidates who used a list to answer this question were able to gain full marks.

Question 7: Compound probability

Candidates showed that they were able to place probabilities in the correct position on the tree diagram and many went on to find the correct probability, gaining full marks for this question. Some candidates did not recognize that addition of two products was required. A mistake that was seen too frequently on candidate scripts was giving probabilities, in part (b), that were greater than 1.

Question 8: χ^2 Test

This question was well answered by the majority of the candidates, many scoring all six marks.

Question 9: Calculus

A surprising number of candidates were unable to correctly expand the expression given in part (a). Most candidates were able to differentiate their function but a considerable number were unable to find the x-coordinate of the minimum point. Candidates must read the questions correctly as answers giving ordered pairs were not awarded the final mark. A number of candidates did not use calculus to determine the local minimum but graphed the function, often achieving full marks for part (c), even when parts (b) or (a) were incorrect or left blank.

Question 10: Use of the graphic display calculator

Many candidates attempted this question but relatively few were awarded the full six marks. Although they were asked to indicate clearly where the graph met the axes, many did not do this. Some entered the functions incorrectly into their calculator. A common error in part (b) was to give ordered pairs and therefore were not awarded the final mark.

Question 11: Geometric progression

The first part of this question was answered quite well, especially by candidates who used a list. Part (b) was poorly answered. Common errors in part (b) were to find the number of rounds rather than the total number of matches played or to take the first term as 512 rather than 256.

Question 12: Quadratic functions and mapping

This question was answered reasonably well with many candidates able to write down the two equations and solve them for a and b. Errors such as mistaking the equation given for $3a^2 + b = 119$ meant that marks were lost even though the candidates appeared to know what they needed to do. Most candidates who were able to set up the equation in part (c) solved it correctly. Follow through marks were awarded to many candidates for correct working with their substituted values from part (b).

Question 13: Financial mathematics

This question appeared to be the most difficult on Paper 1. Some candidates tried to use simple or compound interest formulae to answer part (a) rather than use the financial table. Many candidates forgot to subtract $80,000. Part (b) involved the rounding down of 43,062.6. Relatively few candidates recognized this and many gave a final answer of $43,100.

Question 14: Graphs of trigonometric functions

Candidates were less likely to be able to find the period of the function, although most were able to write down the amplitude. Many could not find the number of points of intersection, although most candidates made an attempt. A common mistake was to list the solutions rather than giving the number of solutions of the two functions.

Question 15: Exponential models

This question was answered very well, although some candidates were not awarded the final mark because the answer was not an integer number of computers. In part (b), some candidates neglected to give their answer correct to the nearest hour and lost the final mark.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to provide working for each question which is worth more than one mark and to write the part number of the question next to the working. Method and follow-through marks are difficult to award if working is hard to read or there is no indication of which part of the question the work relates to. Working must be done within the working box as the IB's online marking does not allow examiners to easily see the whole page at once. Extra pages should not be attached unless they have working on them that is to be marked. The question number and part should be clearly labelled on any extra sheets of paper.

The use of the graphic display calculators is improving but must continue to be emphasized for solving problems, finding points on a graph or checking answers to questions. The calculator should be used across all areas of the syllabus, not just for statistics.

Candidates must be reminded of the difference between: sketch and draw and to note any additional instructions such as indicating intercepts, solving for x or finding the coordinates, exact answers, rounding to three significant figures and giving answers to a specific level of accuracy.

Basic arithmetic skills including order of operations should be stressed, particularly in relation to arithmetic progressions.

Some time in class should be given to practising past examination papers. Most of the questions would have been familiar to candidates if they had answered the many past papers that are published.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 33	34 – 43	44 – 53	54 – 62	63 – 72	73 – 90

The areas of the programme and examination which appeared difficult for the candidates

- Conversion between metric units.
- Probability – without replacement.
- Determining a local minimum.
- Finding the gradient of a tangent.
- Finding the equation of a line in the form $x + by + c = 0$.
- Complex currency conversions.
- The concept of depreciation.

The areas of the programme and examination in which candidates appeared well prepared

- Scatter diagrams and regression lines.
- Percentage error.
- Standard form.
- Venn diagrams and simple probabilities based on a Venn diagram.
- Differentiation.
- Simple exchange rates.
- Simple interest.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Scatter diagram, regression line and percentage error

This question was very well attempted by a significant majority of candidates. Many good and accurate attempts at plotting a scatter diagram were seen in part (a). However, a minority of candidates chose not to use graph paper but instead used their answer book. These candidates achieved, at most, one mark for that part question. Many correct answers were seen in parts (b) and (d) reflecting good use of the graphic display calculator. Whilst many candidates realized that the line of regression passes through the point M , a significant number of candidates seemed to draw their line 'by eye' rather than using the equation found in part (d) and, as a consequence for many, their straight line (or projected line) did not fall within the required tolerances for the second mark. Many candidates understood the requirements for part (f) and full marks were seen on a majority of scripts. Those candidates, however, who used their graph instead scored, at most, two marks here. Many candidates seemed to be well-drilled in giving a suitable reason in part (f) and 'within the data range' or a 'strong correlation' were frequently seen. Percentage error caused very few problems for candidates and many correct answers were seen in part (h).

Question 2: Trigonometry and mensuration

Some candidates assumed that triangle ACB was a right angled triangle with angle ACB = 90°. Such candidates earned no marks for part (a) but were able to recover most of the marks in the remainder of the question. For those candidates who correctly used the cosine rule for part (a), most achieved all 3 marks for this part and used a correct formula for the area of the triangle in part (b) to obtain at least 2 marks for this part. The final mark was not awarded, however, if no units or the incorrect units were given. Parts (c) and (e) were generally well done with many candidates showing the unrounded answer before the required answer. Part (f) proved to be quite problematic for many candidates. Whilst many were able to earn a method mark for $\frac{2190\times2600}{22}$, a significant number of these candidates were unable to convert the units correctly and very few correct answers were seen. Indeed, the most popular answer seemed to be 2590 litres.

Question 3: Venn diagrams and probability

Candidates seemed to be well-drilled in the technique of creating Venn diagrams and using the data from their diagrams to solve problems in probability and this question was well answered. Except for the odd mistake in determining the value of x in part (b), many candidates scored full marks on the first two parts of the question. Indeed, those who calculated an incorrect value of x were able to recover many of the marks in the remainder of the question with the use of follow through marks. ‘Explain in words…’ required candidates to answer part (c) in the context of the question so ‘*E* union *S* intersection not *A*’ earned no marks. Of those candidates who did answer in context, many scored 1 mark for ‘had not visited Asia’ but a significant number used ‘and’ rather than ‘or’ and consequently were not awarded the other mark for expressing $E \cup S$ in words. Whilst many correct answers of 20 were seen for part (d), a significant number of candidates wrote down the incorrect value of 113 which presumably was arrived at by evaluating $n((E \cap S \cap A)')$ rather than the actual demand of the question. Having a Venn diagram seemed to be a good aid for parts (e) and (f) and much good work was seen in these two parts. However, in part (g), a significant number of candidates either chose a “with replacement” method or simply did not know what to do with the probabilities once they were found. As a consequence, this part of the question proved to be quite a discriminator.

Question 4: Functions, calculus and the equation of a straight line

Surprisingly, a correct method for substituting the value of -2 into the given function led many candidates to a variety of incorrect answers. This suggests a poor handling of negative signs and/or poor use of the graphic display calculator. Many correct answers were seen in part (b) as candidates seemed to be well-drilled in the process of differentiation. Part (c), however, proved to be quite a discriminator. There were 5 marks for this part of the question and simply showing that $x = 3$ is a turning point was not sufficient for all of these marks. Many simply scored only two marks by substituting $x = 3$ into their answer to part (b). Once they had shown that there was a turning point at $x = 3$, candidates were not expected to use the second derivative but to show that the function decreases for $x < 3$ and increases for $x > 3$. Part (d) required a sketch which could have either been done on lined paper or on graph paper. The majority of candidates obtained at least two marks here with the most common errors seen being incomplete labelled axes and curves which were far from being smooth. In part (e), many candidates identified the correct coordinates for the two marks available. But for many candidates, this is where responses stopped as, in part (f), connecting the gradient function found in part (b) to the given coordinates proved problematic and only a significant minority of candidates were able to arrive at the required answer of -24. Indeed, there were many NR (no responses) to this part and the final part of the question. As many candidates found part (f) difficult, even more candidates found getting beyond the gradient of L very difficult indeed. A minority of candidates wrote down the gradient of their perpendicular but then did not seem to know where to proceed from there. Substituting their gradient for b and the coordinates $(2, -12)$ into the equation $x + by + c = 0$ was a popular, but erroneous, method. It was a rare event indeed to see a script with a correct answer for this part of the question.

Question 5: Currency conversions, simple interest and depreciation

Despite the fact that “Give all answers in this question correct to two decimal places” was written in bold at the top of the question, many candidates lost one (and only one) mark for giving at least one answer to only a single decimal place. There was a lot of reading in this question and some candidates seemed to lose their way as their solution developed and, as a consequence, lost marks in the latter part of the question. A significant number of candidates obtained nearly full marks for parts (a) through to (d). The marks which tended to not be awarded were not giving the required answer to two decimal places and not adding the amount invested onto the interest earned in part (c). Indeed, many candidates were able to correctly determine the depreciated value of the car on 1st August 2009 by simply finding 91% of the original price. However, part (e) proved to be elusive for many candidates as some simply treated the problem as a ‘reverse simple interest problem’ and subtracted 9% for each of a further 3 years. As a consequence, erroneous answers of the form 17,361.60, from ($27127.50 \times (1 - 0.09 \times 4)$), were often conveniently ignored and rounded to the required answer of 18,600 GBP. Such a method earned no marks at all. There was a lot of information given in the stem to the last part of the question and, as a consequence, many candidates were unable to achieve full marks here. There was certainly a great deal of confusion as to what to divide by 0.8694 (seeing $\frac{18600+8198.05-30500}{0.8694} = -4258.05$ was not uncommon) and even introducing the original exchange rate of 0.7234 caused confusion. As a further example, an incorrect value carried forward from part (c) (1,250.55) led to a negative result. Provided the method was correct (despite an incorrect value carried forward), the three method marks were awarded. However, the negative result of –7,667.53 should have flagged to the candidate that something was wrong somewhere and this could only be in the current part of the question or part (c).

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- Ensure that in mensuration questions where an area (or a volume) is to be determined, that correct units are always given in final answers.
- Identify the number of marks allocated to a part of a question and, using this information, determine the number of processes (method marks) and the number of accuracy marks that are likely to be allocated.
- Show all working to enable method marks to be obtained if answers are incorrect.
- Ensure that answers make sense from the information given. Where a negative answer is found and a positive answer is required (or vice versa), candidates should investigate their method further. Where a final answer is given and the candidate's answer does not give this value or does not round to this given value then the method should be investigated.
- Not cross out their work unless it is to be replaced – crossed out working earns no marks at all.
- Not make assumptions about the properties of a geometric figure where information is not given (see Question 2 (a)).