MATHEMATICAL STUDIES

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 16	17 - 30	31 - 42	43 - 55	56 - 69	70 - 81	82 - 100

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

Range and suitability of work submitted

The vast majority of candidates once again opted for a statistical analysis in an attempt to verify a stated hypothesis. Many candidates produced interesting and sometimes stimulating work but too many others appeared to be doing the bare minimum of work to achieve reasonable levels of achievement. The weakest candidates were clearly making a submission to avoid automatic disqualification from the diploma. Almost all the tasks chosen were appropriate for a Mathematical Studies SL project. There were very few cases where the topic chosen was not an appropriate one and then this was reflected in the analysis part where no, or very few mathematical processes could be applied at all, resulting in more a theoretical project than a mathematical one.

Many of the projects involved questionnaires or surveys but a copy of the questionnaire or survey was not always included with the project. Some candidates did not include their raw data, which precluded cross-referencing of data and checking of mathematical processes.

Many projects, where the student collected their own data, did not describe the data collection process in sufficient detail to allow for the assessment of the quality of the data.

A surprising number of students omitted all simple mathematical processes. In this case the first sophisticated process is considered “simple”. A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering their test invalid. Students also incorrectly drew conclusions about correlation based on their chi-squared test of independence. Few teachers picked up on these mistakes, suggesting that they are either not checking the accuracy of the math processes in sufficient detail when marking or they also do not understand how to correctly perform a valid chi-squared test or they do not understand the assessment criteria for Criterion C well enough.

Many more candidates are relying on technology to do the mathematics for them and often do not do any mathematics themselves. Any mathematical processes using technology only is considered simple.

When using the internet the candidate must remember to include the exact location of the data in the web address in their bibliography.

More and more candidates are producing very short projects which do not reflect the 20 hours allocated for school work plus approximately the same amount of time outside of the classroom.

The range of mathematics that was once seen is now significantly diminished.

However, there were some candidates who produced wonderful projects that achieved high levels in almost every assessment criterion.

It was evident that the guidance provided by the teacher varied from school to school.

The comments made by the teachers on the 5/PJCS forms were very clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against the criteria

Criterion A

The statement of task was usually evident and most candidates described a plan that they would follow but not all of them explained the steps they would be taking or the mathematical processes that would allow them to achieve their goal. It is important to follow the stated plan. If the plan is well documented, then the project is usually better developed and follows a logical structure. Not all plans were well focused. Some projects did not have a title and, as a result, could not be awarded more than 1 mark for this criterion. Some candidates included superfluous information to the extent that their plan became hidden.

Criterion B

Many candidates collected data in a proper and appropriate form so as to facilitate their analysis. However, for other candidates the data was limited in quantity or they did not describe their sample selection and data collection process in sufficient detail to allow for the assessment of the quality of the data. Also, not all candidates set up their data in tables ready for analysis. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted to include this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the tables of data or the mathematical processes used. The candidates must realize that having a lot of data does not always mean that it has the quality needed to gain full marks in this section. It also has to be focused on the task. If data is too simple then it limits the mathematical analysis that the candidate can perform as well as the quality of the discussion on their results. When secondary information is used, candidates must clearly identify the source.

Criterion C

A surprising number of students omitted all simple mathematical processes. In this case, the first sophisticated process is considered “simple”. A large number conducted chi-squared test with insufficient data or non-frequency data, rendering the test invalid. Students also drew conclusions about correlation based on their chi-squared test of independence.

Some candidates only included simple mathematics because their projects did not lend themselves to sophisticated techniques. Many used technology only to perform sophisticated techniques without realizing that this is considered as simple mathematics since the candidate has done no mathematics by hand. Also, it does not show the moderator that the candidate has understood the process. Some candidates introduced mathematical processes that were totally irrelevant. This can actually result in the candidate losing marks.

Teachers must check the accuracy of their students’ mathematical processes before awarding an achievement level in Criterion C.

Criterion D

Most candidates produced at least one result that was consistent with their analysis. Few candidates produced thorough explanations of what had been found, calculated and observed. Often this was because the project was too simple and comprehensive discussion was not possible.

The stronger candidates did a good job of presenting partial conclusions as they went along and then summarized these to give an overall conclusion at the end. In some projects there was a tendency to provide justification or explanation for the results found which were often based on personal assumption or hypothetical reasoning.

Criterion E

More candidates are now commenting on validity.

Their discussions generally centre on data collection. Less often was a student able to comment on the validity of the processes themselves. Some candidates are beginning to add sensible suggestions for extensions of their project.

Criterion F

Most of the projects were structured and communicated well. Many candidates recorded their actions at each stage. It is also important to ensure that the notation and terminology is correct otherwise this restricts the mark that can be awarded for this criterion. Many candidates lost marks this session due to errors in either notation or terminology. Some candidates do not seem to be aware that calculator/computer notation is not always correct mathematical notation.

Criterion G

The majority of teachers appear to have awarded marks appropriately. However, some schools really abused this mark, giving it to all students when the quality of work was very simplistic and hastily done.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Give them examples of "good" projects so that they know what is expected of them.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the students the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology, remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage students to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos and computer notation.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them.
- Emphasize the importance of meeting deadlines.
- Inform students about sampling techniques.
- Remind them to include all raw data either in an appendix or as part of the task.
- Show students how to use Equation editor or Math Type.
- Remind them of the importance of including simple mathematical processes in their projects.
- Check the calculations in each project.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals and set interim deadlines to monitor the progress of the project and catch any major deficiencies in good time.
- Write a comment to justify each achievement level awarded.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 40	41 - 52	53 - 65	66 - 77	78 - 90

General Comments

The paper appeared to be of appropriate length and difficulty. Indeed, where candidates made no response to a question this was invariably due to the degree of difficulty of the question rather than a time issue. The comments on the G2 forms were appreciative of the syllabus coverage, of the level of difficulty, and diversity of questions.

The areas of the programme and examination which appeared difficult for candidates

- Significant figures where there is at least one leading zero after the decimal point (Q1)
- Use of the GDC in correlation and regression questions (Q4) and standard deviation (Q2)
- Identifying the required area of a Venn diagram from a compound set notation expression (Q5)
- Compound probabilities (Q10)
- Simple and compound interest (Q11)
- Quadratic functions and curve sketching (Q13)
- Decreasing (increasing) functions (Q14)
- Exponential functions (Q15)

Five of these areas identified correspond to the latter questions on the paper which are designed to be more challenging for candidates.

The areas of the programme and examination in which candidates appeared well prepared

Despite the challenges of the last third of the paper, the majority of candidates did very well on the first eight questions. Much good work was seen in number work and whilst some candidates seemed challenged with the correct use of the GDC, many scored reasonably well on Q4. The responses to the logic question seemed to be somewhat better than in previous years as the majority of candidates scored well on Q3. Despite the difficulties with the last part of Q5, a significant majority of candidates scored at least 4 marks on this question. Much good work was also seen on Q6, Q7, Q8 and Q12, suggesting that candidates are well drilled in sequences, currency conversion and coordinate geometry. Of the remaining questions, most were quite challenging but allowed the more able to confidently show what was required.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Standard form, significant figures and decimal places

Although the use of radians leading to an incorrect answer of -0.1785774388 was seen on a minority of scripts, many candidates produced correct answers for parts (a) and (b)(i). The requirement for an answer to 3 significant figures led many to count the first zero after the decimal point and as a consequence gave an incorrect answer of 0.094. Despite any previous incorrect working, it was pleasing to see that most candidates were able to express their answer to part (a) in standard form.

Question 2: Statistical measures

Whilst many knew what the mode was, there was some confusion by weaker candidates who interpreted the required value as the maximum value in the list, namely 7. Many candidates gave the correct value of the mean in part (b) but a surprising number seemed to have selected the incorrect value from their calculator display for the standard deviation. In mathematical studies, the smaller value of the standard deviation, σ_x , should be given. In part (d), ordered lists were often seen but it proved problematic for a significant number of candidates to find Q_1 and Q_3. Common mistakes such as $Q_3 = \frac{6+6}{2} = 6.5$ and $Q_1 = 3$ were seen on many scripts.

Question 3: Logic

Weaker candidates had some difficulty here with the majority scoring less than 2 marks on this question. The more confident candidates were able to score well with most marks being lost only on completing the truth table for $(\neg p \veebar q)$. As a consequence, the final column entries of the table were often incorrect but earned the (A1)(ft) mark. Many candidates went on to correctly identify the correct (ft) response to (b)(i) and were able to support their answer with a correct reason.

Question 4: Correlation and regression

Able candidates scored very well on this question showing good use of their GDC and marks in excess of 4 were achieved by a large majority of these candidates. It was clear however, from some responses, that either the topic had not been taught or had been poorly understood by candidates and few, if any, marks were achieved by such candidates. In part (b), many candidates quoted a correct regression line equation using $\bar{x}, \bar{y}, s_{xy}$ and s_{x^2} but then seemed to be at a loss as to what to do with it.

Question 5: Sets and set notation

Much good work was seen in parts (a) and (b). However, there was much confusion in candidates’ responses to part (c) as many could not determine the required answer where a union was involved with a complement. The result was that either candidates simply ignored

$n[(A \cup B)']$ and evaluated n(A) = 21 or ignored $n[(A \cap B)]$ and evaluated $n(B') = 18$. Irrespective of ability, the modal mark for this question was four with very few candidates achieving more than this mark.

Question 6: Arithmetic sequence

At whatever ability, there were good attempts by all candidates on this question with an overwhelming majority scoring half marks or more. The most common error was in part (a) where 5, rather than -5 resulted in a lost mark. Recovery was, of course, possible in the remainder of the question. Further errors occurred where lists, rather than formulae, were used in parts (b) and (c). Using properly constructed and accurate lists were not in themselves penalized; arithmetical errors seen in a significant number of lists given by candidates, however, were penalized.

Question 7: Coordinate geometry

Overall, there was a very good response to parts (a) and (b) with only a few candidates giving an incorrect expression for the gradient in part (b). Occasionally, the final mark in part (b) was lost because the negative sign was dropped by some candidates. Many able candidates recognized they needed to do something with the equation $y = mx + c$ in part (c). Weaker candidates clearly showed a lack of understanding of an equation of a line and either simply gave a numerical answer for this part of the question or tried to use the coordinates of *M* into what they believed was the required equation of the straight line. A popular incorrect answer seen was $y = 3x + 5$.

Question 8: Currency conversion

At whatever ability, there were good attempts by all candidates on this question with an overwhelming majority scoring half marks or more. Indeed, three out of the first four marks were invariably earned with only answers of 7.2 EUR losing the final mark in part (b). In part (c), errors were invariably caused by candidates ignoring the commission charge or multiplying by 0.8202 rather than dividing by this value. Another common, but incorrect, method seen was multiply 152.80 by (1 + 1 – 0.8202), giving the wrong answer of 180.27 USD. Both marks were lost in all the cases listed.

Question 9: Areas and volumes

This question was only done well by the more able candidates. For the lower quartile of candidates, about a half of these scored no more than one mark in total for this question. In many cases, formulae were either misquoted or misused. Indeed, in part (a) many ignored the hemisphere, choosing to use the formula for the volume of a sphere instead. Some candidates who correctly used a hemisphere in part (a) then treated the surface area as a sphere in part (b). A minority of candidates thought the area to be painted in the last part of the question was a circle rather than a hemisphere.

Question 10: Probability

The vast majority of candidates were able to pick up the first two marks by confidently identifying the *number of favourable outcomes/total number of outcomes*. Difficulties arose however when combining events and only the more able candidates were able to progress successfully with the remainder of the question. As usual in this type of question, there was an abundance of incorrect answers greater than 1 given.

Question 11: Simple and compound interest

This question was either done very well or very badly. Most of the problems focused on the use of 18 months in both parts of the question and the interpretation of 15% **compounded monthly** in part (b). As a consequence, 540000 INR was a common incorrect answer given in part (a) earning no marks for this part of the question. In part (b), some candidates simply worked out the interest earned. If a correct substitution into the correct formula was seen leading to 50115 then only one mark was lost. Whilst many correctly quoted formula were seen in part (b), an incorrect substitution (particularly poor or missing use of the factor 12) lost the next two marks and, whilst the final mark could be earned irrespective of incorrect working, many candidates either ignored the final demand or did not know how to give their answer to the nearest rupee.

Question 12: Geometric sequence

The upper quartile of candidates scored well on this question with the vast majority scoring more than 4 marks. However, the lower quartile did very badly with the majority scoring less than 2 marks. A fundamental error in part (a) resulted in many less able candidates using a common ratio of 4 instead of ¼. Where lists were used in either part of the question, they were either invariably incomplete or contained numerical errors. Indeed, using lists seem to be as problematic in this question as they were in Q6 on arithmetic sequences. Correctly quoted and substituted formula in a correct inequality (= sign was allowed) did earn many candidates two marks here. The required answer of 7 however did not always follow.

Question 13: Quadratic function and curve sketching

This question was not answered well at all except by the more able. Indeed, of the lower quartile of candidates, the maximum mark achieved was only 1. Of those that did make a successful attempt at the question, very few used the fact that $1.25 = -\frac{k}{2(2)}$ preferring instead to differentiate and equate to zero. But such candidates were in the minority as substituting $x = 1.25$ into the given quadratic and equating to zero produced the popular, but erroneous, answer of -5.7. Recovery was possible for the next two marks if this incorrect value had been seen to be substituted into the correct quadratic, along with $x = 1.25$ to arrive at an answer of 0.This would have given (M1)(A1)(ft). However, candidates who had an answer of $k = -5.7$ in part (a)(i), invariably showed no working in part (ii) and consequently earned no marks here. Irrespective of incorrect working in part (a), the quadratic function clearly passes through (0, 4) and has a minimum at $x = 1.25$. Using this information, a minority of candidates picked up at least one of the two marks in part (b).

Question 14: Calculus and decreasing function

This question was quite a good differentiator with many able to score at least one mark in part (a). Part (b) proved however to be quite a challenge as many candidates did not seem to understand what was required and were unable to use their answer to part (a) to help them to meet the demands of this question part. The top quartile scored well with virtually everyone scoring at least three marks. The picture was somewhat reversed with the lower quartile with the majority of candidates scoring 2 or fewer marks.

Question 15: Exponential function

This was perhaps the most difficult question on the paper. Being the last question some candidates may have felt that they were under pressure to complete and many scripts showed no attempt at an answer to this question. The response by the upper quartile of candidates was quite encouraging with many achieving at least 4 of the 6 marks available. For the rest, many fell at the first hurdle and were unable to obtain a value of k. This, in turn, led to problems in finding c. For a large number of candidates the only mark that they achieved was identifying that the asymptote was a linear equation in y.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- give all numerical answers to three figure accuracy or the exact value answer where no accuracy is requested in a question. Where a three figure answer is produced, it is helpful to precede this with the calculator displayed value.
- critically examine their answers to see whether or not they are sensible in the context of the problem set - e.g. probability answers greater than one should always be questioned.
- show all working to enable method marks to be obtained if answers are incorrect. Many A marks for second, or subsequent parts to a question are often follow through marks. But, these can only be awarded if working is provided to support the answer, however trivial.
- not cross out their work unless it is to be replaced - crossed out working earns no marks at all.
- use the GDC more effectively - understand all the relevant functions and use of GDC. Candidates should learn to use their GDC in diverse contexts - graphing functions, finding points of intersection of two graphs, solving simultaneous equations, using it with statistics questions.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 37	38 - 48	49 - 59	60 - 70	71 - 90

General Comments

The great majority of candidates attempted each question and were able to gain some success throughout the paper. However, as usual, there were a number of centres that had not prepared their students properly for the examination; omitting topics; the notable areas being the chi-squared test and the differential calculus. Time seemed not to be an issue for candidates. Rather, the difficulty of the paper seemed to be the sole discriminator. A pleasing number of candidates were able to display their knowledge and skills over the entire paper, achieving marks in the high eighties (as a percentage score). Clearly many centres are doing fine preparatory work. The examination was felt to be an appropriate test of the syllabus by the great majority of teachers who submitted G2 forms. There were a few concerns raised about clarity of wording compared to the May paper and also that the paper was more of a challenge.

It was commented upon that the coverage was overly biased towards statistics. However, the entire syllabus was covered if one considers both papers as a single examination. There will be variation from examination to examination; thus confirming the need to address the entire syllabus.

Many candidates lost marks in the “show” part of question 4. However, it was pleasing to note the degree of rigour that the majority showed in completing the proof in 3(d). Once again it must be reiterated that when candidates are required to reach a given answer to a specified accuracy as in 2(c)(ii), they must first write down the value they obtain correct to a higher degree of accuracy **and** then write down the given value so that these can be compared.

The areas of the programme and examination which appeared difficult for candidates

- The variables in the formulas of trigonometrical functions - relating these to amplitude, period etc
- Formal differential calculus and its applications to tangents and normals
- Drawing a graph to the correct scale
- The more complex aspects of probability theory

The areas of the programme and examination in which candidates appeared well prepared

- Use of sine and cosine rules and the associated area formula.
- Chi-squared test using the GDC

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Cumulative Frequency Diagram

The great majority of candidates found this question to be a good start to the paper. The different class widths seemed to cause more problems to the teachers commenting on the G2 forms than to the students. However, 1(b) was a discriminator at the grade 4 level. Most candidates were successful in drawing the cumulative frequency curve or attempting to do so. There were a small number who clearly had never had any experience with this type of graph.

A common error was the incorrect plotting of points at the interval midpoints. Weaker candidates plotted bar charts.

A small number of candidates did not use the graph paper provided, preferring instead to use lined paper. This is to be strongly discouraged since no judgment will be made about the scale used or the accuracy of the plotted points. Similarly, the graph will not be used to benefit students whose answers lie outside accepted tolerances but who have shown working. Drawing an accurate graph requires the use of graph paper.

Question 2: Probability and Contingency Table

The simple probabilities beginning this question were successfully attempted by the great majority. Most errors in the latter parts occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table. Probability questions in this course are, in the main, contextual and the reliance of formulas is not always beneficial to the candidates. Only the best candidates realized the significance of part (b) as a link to the chi-squared test.

This was well attempted by the majority, the weakness being the sole reliance of the calculator to calculate expected value. However, there still remains confusion between critical and *p*-values as the basis for accepting the null hypothesis.

Question 3: Trigonometry

The vast majority of candidates scored very well on this question. Those who did not attempted it using the trigonometry associated with right angled triangles. There were few problems with the use of radians and part (d), which was expected to prove challenging, was successfully overcome by more than half of the candidature. Problems arose mainly because of a lack of clarity in identifying the correct triangle.

Question 4: Differential Calculus

As usual, the content in this question caused difficulty for many candidates. However, for those with a sound grasp of the topic, there were many very successful attempts. The curve was given so that a comparison could be made to a GDC version and the correct form of the derivative was also given to permit weaker candidates to progress to the latter stages. Unfortunately, some decided to proceed with their own incorrect versions, in which case **very limited follow through accrued**. It should be emphasized to candidates that when an answer is given in this way it should be used in subsequent parts of the question.

As in previous years, much of the question could have been answered successfully by using the GDC. However, it was also clear that a large number of candidates did not attempt either to verify their work with their GDC or to use it in place of an algebraic approach.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested. Some centres still do not teach the differential calculus.

Question 5: Trigonometric Function (The Ferris Wheel)

Most candidates were able to start this question. Those of an average ability completed it to the end of part (c) and the best gained good success in the latter parts. Its purpose was to discriminate at the highest level and this it did. The final part, linking basic trigonometry to the wheel proved challenging to all, but there were candidates who were able to complete this or to link the diagram of the wheel to the equation of the curve.

Some concerns were raised on the G2 forms as to the appropriateness of this question. However, the MSSL course tries in part to link areas of the syllabus to "real-life" situations and address these. A look back to past years' examination papers, and to the syllabus documentation, should yield similar examples.

Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently
- Ensure candidates draw graphs on graph paper
- Do not ignore the theory behind the chi-squared test
- Time management - a mark a minute is the guide - and attempt all questions
- Ensure candidates use given results in the latter part of questions
- Teach the whole syllabus; all areas will be examined in some form - in either Paper 2 or Paper 1
- Practice with "show that" questions; candidates must communicate through their mathematics
- Ensure candidates start each question on a new page and show their working