# MATHEMATICAL STUDIES

## Overall grade boundaries

### Standard level

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 – 16 | 17 – 29 | 30 – 43 | 44 – 56 | 57 – 69 | 70 – 82 | 83 – 100 |

## Standard level project

### Component grade boundaries

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 – 4 | 5 – 6 | 7 – 8 | 9 – 11 | 12 – 14 | 15 – 16 | 17 – 20 |

## Range and suitability of work submitted

This session even more projects than usual were of a statistical nature and very few other topics were seen. There was also a wide range in the standard of the work submitted. The most successful projects were those where the candidate had a strong personal involvement. The weakest candidates were clearly making a submission to avoid automatic disqualification from the diploma. However, the average grade for the project was higher than in the previous session which implies that the average candidates and the stronger candidates were performing better than in the previous session.

Many of the projects involved questionnaires but a copy of the questionnaire was not always included with the project. In some cases the candidate did not include any raw data, making it impossible to check the calculations.

Many more candidates are relying on technology to do the mathematics for them. Any mathematical processes using technology only must be considered simple because the candidate has not performed any mathematics themselves. When a process has been performed using technology and not commented on then the moderator does not know if the candidate has really understood what they were doing.

When using the internet the candidate must remember to include the web address in their bibliography. More candidates are now including a bibliography.

The comments made by the teachers on the 5/PJCS forms were very clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

## Candidate performance against the criteria

A. The statement of task was usually clear and most candidates described a plan that they would follow. It is important to actually follow the stated plan. If the plan is well documented, then the rest of the work flows from it. Candidates with clear statements of task and plan tended to be able to extract more depth from their projects because they knew what they were looking for. Not all plans were well focused. Some projects did not have a title.

B. The data collection varied from 2 pieces of data to well over 100 pieces. The candidates must realise that having a lot of data does not always mean that it has the quality needed to gain full marks in this section. It also has to be focused on the task. The candidate must make sure that the data collected is relevant to their project. Most of the candidates collected their data and set it up in tables ready for use. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted to include this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the mathematical processes used. A large number of candidates just downloaded data straight from the internet with little thought being given as to how much of that information was really relevant to their task. It is also important to state the website in the bibliography. If data is collected “at random” then the candidate needs to explain what “at random” means in their project.

C. Some candidates only included simple mathematical processes in their projects. Many relied solely on computer generated results without explaining the techniques. The range of mathematics applied was restricted in some projects. Some candidates introduced mathematical processes that were totally irrelevant. When a scatter diagram indicates that there is no correlation between two variables then it is meaningless to go on and calculate the correlation coefficient or line of best fit. Also working out standard deviations without having a meaningful discussion on what the results indicate is of no value. This can actually result in the candidate losing marks. In some cases this was the only mathematical process attempted and not all candidates appeared to understand what they were doing.

D. Most candidates produced results that were consistent with their analysis but often these were rather brief. Few candidates produced detailed discussions. In many cases the conclusions were obvious and not very thorough. Some candidates gave subjective reasons for results found that were unrelated to any mathematical processes performed.

E. More candidates are now commenting on validity. Usually this is more to do with the process of data collection than anything else. Unfortunately only a few of the candidates also commented on the mathematical processes but, these candidates managed to do this thoroughly. Some candidates are beginning to add sensible suggestions for extensions of their project.

F. On the whole, the projects were well structured. Many candidates recorded their actions at each stage. It is important to ensure that the notation and terminology is correct. Many candidates lost marks this session due to errors in either notation or terminology. Candidates must be careful not to use computer or calculator notation.

G. The majority of the teachers appear to have awarded marks appropriately.

## Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Ensure that students select a topic which is suitable for analysis and of interest to them
- Emphasise the specific purpose of the project
- The project exercise should be introduced at an early stage in the course to avoid rushed and often poor work handed in just to satisfy a requirement
- Give them examples of "good" projects so that they know what is expected of them
- Make sure that they are aware of (and understand) the assessment criteria
- Write full and clear comments on the 5/PJCS form
- Stress the importance of using appropriate mathematical notation
- Give candidates a second chance to correct errors
- Stress the significance of collecting sufficient data
- Put raw data into an appendix
- Encourage them to think up their own task and explain the plan thoroughly
- Tell them to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables
- Check that the mathematics used in the project is relevant
- Encourage the candidates to use both simple and sophisticated mathematics
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator
- Encourage candidates to pay more attention to detail such as labels on axes
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the original work of the candidate to the moderator
- Meet with the candidates at regular intervals to monitor the progress of the project

- Show comments and corrections on the projects and check the candidates' calculations

# Standard level paper one

## Component grade boundaries

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 – 12 | 13 – 24 | 25 – 38 | 39 – 51 | 52 – 63 | 64 – 76 | 77 – 90 |

## General Comments

The paper appeared to be of appropriate length and difficulty. The comments on the G2 forms were appreciative of the syllabus coverage, of the level of difficulty, and diversity of questions.

## The areas of the programme and examination which appeared difficult for candidates

It appears that it is difficult for the candidates to read graphs and diagrams, interpret them, and draw conclusions. This paper presented such cases with questions 4, 5, 10, 13, and 15. It also appears that candidates have difficulty with using their graphic display calculator for calculating mean and standard deviation, and solving simultaneous equations. Students have also struggled with articulation of a reason to justify a true or false statement. Other areas of difficulties were: writing a contrapositive of a statement, finding a probability of a combined event, finding the volume of a prism, finding an expression for the area of a shaded region, identifying the range of a function and writing the equation of the horizontal asymptote to a given graph.

## The areas of the programme and examination in which candidates appeared well prepared

The majority of the candidates showed good time management skills and very few questions were left unattempted. Almost all students were able to find the volume of a cuboid and the percentage error. The questions concerning arithmetic and geometric sequences were also answered completely or at least partially by almost all students. Most candidates were able to demonstrate good knowledge of logic, standard form of numbers, and gradient of a line. Very few students lost marks to unit penalty. Working was shown by the majority of the candidates so that follow-through marks and method marks could be awarded when final answers were incorrect. Most scripts were neatly presented.

# The strengths and weaknesses of candidates in the treatment of individual questions

**Question 1: Numbers in standard form**

In general this question was answered correctly by the majority of the candidates. Part b presented difficulty for some students by asking them to compare the given numbers. A common error found in this part was that the value of π was given as 3.14. A method mark was awarded when a comparison was attempted.

**Question 2: Logic**

This question was well answered with most candidates able to complete the truth table correctly in part a) and write the correct compound proposition in symbolic form in part b). A significant number of candidates could not write the correct contrapositive, although most were awarded one mark for writing an implication.

**Question 3: Volume, and percentage area**

This question proved to be the one that most candidates answered correctly. Many received full marks and the only error seen was incorrect substitution in the percentage error formula.

**Question 4: Matching graphs of different linear equations with given values for *m* and *c***

Many candidates received full marks and a number received 3 marks for giving two correct answers. Very few candidates were awarded zero marks. As most candidates did not show working for this question it is difficult to comment on the errors that might have been made.

**Question 5: Box and whisker plot**

Many students had difficulty with reading the box and whisker plot and interpreting this question. Some candidates had difficulty with finding the range in part (a)(iii). Many wrote down the end points for the required range of data instead of writing the difference between the largest and smallest values. A number of candidates had problems estimating the number of students who achieved a mark greater than 32. Many students used the number 40 instead of the total number of student 56 for the estimation in part b).

**Question 6: Probability**

This question was generally well answered by many of the candidates. Many found the conditional probability in part b) easier compared to previous sessions, since they were able to write it down directly from the table. A number of candidates found the final part difficult with a significant number unable to use the combined events probability formula correctly.

**Question 7: Statistics**

This question was not well answered. Many candidates could not find the mid interval value, and used their graphic display calculator incorrectly to find the mean and standard deviation.

The candidates who showed working and correct method in part c) were awarded the final two marks. If working was not shown then these marks could not be awarded.

**Question 8: Arithmetic sequence**

Most candidates achieved at least the first three marks on this question and a significant number gained full marks. Up to five marks could be awarded even to students who did not find correctly the value of the *n*th term, but showed correct method and attempted to find the value of n. Some candidates lost the last mark for giving a non integer value.

**Question 9: Sets and set notation**

The question was not well answered by the majority of the candidates. Many did not identify the universal set correctly and so took 3 to be a member of this set. This affected their answers in a)(i) and a)(ii). Not many students answered (b) correctly. Some listed all correct elements of the given set instead of just one, which shows that they did not read the question carefully. Although many candidates could indicate which statement in the table in c) was false, often they were unable either to identify or articulate a correct reason for it.

**Question 10: Quadratic function**

A number of candidates left out this question which indicated that this topic was either entirely unfamiliar, that this topic of the syllabus had perhaps not been taught, or was barely familiar. A few candidates wrote down coordinate pairs when asked for a solution to the equation. A number of candidates wrote down the formula for the equation of the axis of symmetry without being able to substitute values for *a* and *b*. When given the minimum value of the graph a small number of candidates could identify the range of the function correctly. Overall this question proved to be difficult with its demands for reading and interpreting the graph, and dealing with additional information about the quadratic function given in the different parts.

**Question 11: Geometric sequence**

Parts a) and b) of this question were well answered by many of the candidates, although in some cases students wrote ½ instead of 2 for the common ratio in a). Many candidates were also able to set an equation in c) with a correct expression of the sum of the first n terms equated to 31.9375, for which they gained two more marks. The last mark in many cases was not awarded either because the candidates didn't know how to solve the equation or/and gave an incorrect answer.

**Question 12: Area and volume of a hexagonal prism**

This question proved to be difficult for a number of candidates. Most were able find the size of the angle in part a), but many had problems finding the area of the triangle in part b). A significant number of candidates were unable to use the Pythagoras Theorem correctly to find the height of the triangle AOB. Those who used the formula for the area of a triangle $A = \frac{1}{2}ab\sin C$ were more successful in this part of the question. It was surprising that a great number of candidates were unable to find the volume of the prism-- many incorrectly used the formula for calculating volume of a pyramid rather than a hexagonal prism.

**Question 13: Identifying coefficients of a function from its graph**

A significant number of candidates found it difficult to identify and write two equations that relate *p* and *q*. Many of those who wrote the equations were unable to solve them or use their GDC to find the values of *p* and *q* in part b). Although the question in part c) was quite standard, there were many errors in the responses. Many students wrote $x = 2$ or only 2 instead of $y = 2$.

**Question 14: Area and perimeter of a given region**

Some candidates were able to answer this question correctly, but the majority experienced difficulty in finding the correct expression for the area of the shaded region. Those who showed working could then be awarded follow through marks for correctly equating their expressions to the given area and for their found value of *x*. Many candidates also could not find the perimeter of the shaded region in part c) even though they had found the value of *x* correctly.

**Question 15: Trigonometric function**

Most candidates attempted this question, which shows that exam time was sufficient. Part a) required that the candidates draw the graph of the function (rather than sketch it) and so an accurate graph was expected. Most of those who drew a correct graph in a) were able to answer correctly b) and c). Many candidates were also able to answer part d) and could use their graphic display calculator to find the values of *a* and *b*.

## Recommendations and guidance for the teaching of future candidates

- Read the questions carefully - it is essential that candidates read the exam carefully and know exactly what questions they have to answer.
- Pay attention at the command terms in a question - candidates should read the command terms in a question carefully. Highlighting the command terms may help to keep the types of actions that need to be taken in sight.
- Show working - all relevant working should be shown in each question with the question part indicated in the working box. Follow through marks can then be awarded where appropriate.
- When showing working, label the part of the question you are answering - proper labelling is necessary as much to help your quick review at the end of the exam as for the examiners when reviewing and marking your work.
- Use GDC more effectively - understand all the relevant functions and use of GDC. Candidates should learn to use their GDC in diverse contexts—graphing functions, finding points of intersection of two graphs, solving simultaneous equations, using it with statistics questions. There is no need to explain how the GDC was used, i.e. which keys were pressed, etc.

- Give your answers to the required accuracy - all answers should be given to the accuracy required by the question, or otherwise to 3 significant figures. Answers that do not satisfy the required accuracy are penalized.
- Check answers carefully - candidates should be reminded to check their answers to ensure they are reasonable in the context of the question.
- Review Past Papers - candidates should familiarize themselves with previous papers, their format, and key terms that are used.
- Work on diverse questions - it is necessary that candidates have experience with diverse questions. It is important for them to work as much with straightforward questions as with ones that require more interpretation and analysis of the given information and the utilization of problem solving strategies.

# Standard level paper two

### Component Grade Boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 – 14 | 15 – 28 | 29 – 40 | 41 – 51 | 52 – 62 | 63 – 73 | 74 – 90 |

## General Comments

The examination was deemed to be an appropriate test of the syllabus by the majority of teachers submitting G2 forms. Most of the teachers found this exam to be of a similar standard compared to previous sessions. This paper differentiated the candidates well as the range of marks varied from very few marks to full marks. Time did not seem to be a factor for the majority.

A number of candidates lost marks in the “show that” parts of the questions. When candidates are required to reach a given answer that is written to a specified accuracy, they must write down that value with a higher degree of accuracy (unrounded value). Further, premature rounding resulted in marks being lost.

In the questions asking for angles it is becoming far less common to find candidates using their GDC in radians due to GDC reset; this is an encouraging trend. Although follow through marks were awarded, in some cases marks were lost because of being either negative or unrealistic.

Many candidates appeared to be well prepared giving their answers to the correct number of significant figures or to the specified accuracy in the financial questions where appropriate, and using the correct units. However many others were not aware and were penalized with all three penalties, accuracy, unit and financial leading to the loss of three marks in the paper.

Despite incorrect answers, follow through marks were given when proper working was shown.

## The areas of the programme and examination which appeared difficult for candidates

- Describing in words the elements of a set
- Combining probabilities
- Equation of a line in the form $ax+by+d=0$ where $a,b,d\in\square$
- Finding time given average speed and distance
- Finding the rate for simple interest
- Finding the coordinates of the points at which the tangent touches the curve

## The areas of the programme and examination in which candidates appeared well prepared

- Venn diagrams
- Chi-squared test
- Gradient of a line
- Currency conversions
- Differentiation
- Sketching a graph

## The strengths and weaknesses of candidates in the treatment of individual questions

**Question 1 Part A: Venn diagrams – Probabilities**

This question was in general well done. Candidates began the paper well by drawing the Venn diagram correctly. Some students omitted the rectangle (universal set) around the three circles. There were quite a few errors in (c) as some students forgot to convert their answers to percentages. Also describing in words what the students in $X\cap Y'$ had for breakfast seemed to be difficult for the majority of the candidates. Some misread what $Y$ was and even more missed the complement sign. However, the main problem in answering this question seemed to be the lack of knowledge in the relationship between set theory and logic (use of "and" and "or"). Combining probabilities caused problems to many. Common wrong answers were $\frac{10}{100}$, $\frac{10}{100}\times\frac{10}{100}$ or $\frac{10}{100}+\frac{9}{99}$.

**Question 1 Part B: Chi-squared test**

In general this part question was well answered. The major concerns of the examining team were the following:

- In (f) many students wrote down the expected values table (from the GDC) and highlighted the correct expected value, 12.6. As this is a "show that" question the use of the GDC is not expected and therefore no marks are awarded for this working. Instead it is expected the use of the formula for the expected value with the correct substitutions.
- In (e) surprisingly many candidates found the $x^2_{calc}$ through the use of the formula. Unfortunately this led to some incorrect answers and also to a bad use of time. The question clearly says "use your graphic display calculator" and it is worth 2 marks therefore a student should not spend more than 2 minutes to answer this part question. Time management is essential in this type of examinations and the IB rule is one minute – one mark.

**Question 2: Coordinate geometry**

This question was in general well answered. In part (a) the gradient of the line AB was correctly found although some candidates did not substitute well in the gradient formula and found answers as $\frac{1}{2}$ or -2. Also some students read B as (0, 8) instead of (0, 9). In part (b) many students again did not make good use of time as they found the equation of the line instead of just extending it to find the $y$ - intercept. The equation of $L_2$ in (c) was correctly found in the form $y = mx + c$ but very few students were able to rearrange the equation in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. In (d) many candidates found the coordinates of point D by solving simultaneous equations which led again to a waste of time. The last two parts of this question were well done by those students that attempted them.

**Question 3: Trigonometry**

Many students lost the accuracy penalty mark (AP) and the unit penalty mark (UP) in this question. Also very few students forgot to reset their calculator to degrees and therefore gave their answers in radians. The weak students answered parts (a) and (b) using right-angled trigonometry. Different types of mistakes were seen in (a) when applying the cosine rule: some forgot to square root their answer and others calculated each part separately and then missed the 2 minuses. Part (b) was better done than (a). Follow through was applied from (a) to (c). Part (d) was not well done. Most of the students lost one mark in this part question as they did not show the unrounded answer (2.0550...). Part (e) was fairly well done by those who attempted it. In (f) there were very few correct answers. Students found it difficult to find the time when the average speed and distance were given.

**Question 4: Financial math**

Most of the students were penalized in this question for not given their money answers correct to the specified accuracy (2 decimal places).

**Part A: Currency conversion – commission**

The first three parts were well done. Some students gave their answer to part (d) in (e) and their answer to (e) in (d). This means that when reading commission they directed their answers to a percentage (commission rate).

**Part B: Compound and simple interest**

Most of the students used the correct formula but not all made the correct substitution. From those that made the correct substitution, very few showed the unrounded answer. Part (b) was well done. In part (c) the majority did not put the interest (only) in the formula but the total amount $\$1109.70$.

**Question 5: Calculus**

The value of $f(0)$ and the derivative function, $f'(x)$ were well done in parts (a) and (b). In part (c) many candidates found $f(1)$ instead of $f'(1)$. In part (d) many students did not use their $f'(x)$ to find the *x*-coordinates of M and N and instead used their GDC. The sketch was generally well done although some students forgot to label M and N or did not use the specified window. The last part of the question was a clear discriminator. Examiners were pleased to see how this challenging question was solved using alternative methods.

## Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently, especially with graphs of functions and statistics
- Time management – a mark a minute is the guide – and ensure that all questions are attempted
- Cover the whole syllabus; it will all be examined
- Practice with "show that" questions by having candidates communicate through their mathematics
- Ensure candidates label and scale the axes whenever they draw or sketch a graph
- Ensure candidates start each question on a new page and to show all their working
- Formula booklet should be part of everyday teaching so that candidates become familiar with it
- Applying APs, UPs and FPs should be part of a teacher routine in marking throughout the 2 year course
- Train candidates to write an appropriate amount of detail in their responses. Both too little and too much are not good options
- More time should be spent on algebra to help students to improve their algebra skills
- Candidates should not write down which GDC keys they used to find an answer; marks will not be awarded for it

- Show candidates the importance of starting each question on a new page and to show all their working