# MATHEMATICAL STUDIES

## Overall grade boundaries

### Standard level

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 14 | 15 - 25 | 26 - 39 | 40 - 53 | 54 - 67 | 68 - 80 | 81 - 100 |

## Standard level project

### Component grade boundaries

| Grade: | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Mark range: | 0 - 4 | 5 - 6 | 7 - 8 | 9 - 11 | 12 - 14 | 15 - 16 | 17 - 20 |

## Range and suitability of work submitted

Almost all the tasks chosen were suitable for a Mathematical Studies project. Many candidates had taken pride in their work and produced very good pieces of work that had been carefully checked by their teacher. Others appeared to be rushed, were far too short or unfinished, contained many errors and obviously did not use the 20 hours class time that should be allocated to the project. In some projects the topics were too descriptive having little or no mathematical content in them. In some schools the teacher had obviously written up a template for the students to work from because all the projects contained the same type of data collection and the same mathematical processes. The individuality of the student is lost in cases like this.

The majority of the projects had a statistical basis. The strongest candidates undertook an experiment and were able to include experimental design in the planning stage. As a result their discussions of validity were strong as they could comment on any problems in the process of carrying out the project. Also they were able to compare their experimental results with those published on the internet and hence could comment in a meaningful way about the conclusions made. The weakest candidates had massive flaws in their projects.

The majority of the projects were well presented with few this year being hand written. Also, the project must not simply be a piece of work with lots of calculations. There must be a flow of ideas between the different sections and the interpretations of the results generated need

to be clear and concise. Graphs must be clearly labelled, pages numbered and all sources documented.

A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the calculations. Some of the candidates did not have any calculations at all and just showed results from their calculator or computer with formulae nowhere to be found. This has the result of leaving the moderator to wonder whether or not the candidate really understood what they were doing.

When using the internet the candidate must remember to include the web address in their bibliography.

## Candidate performance against the criteria

A. Most of the topics had a title. Many candidates who undertook a survey or collected data from the internet gave no indication as to how the particular individuals were selected or how the countries studied were chosen. The majority of the candidates stated their task but there are still some candidates who find it difficult to explain this in a clear and concise way. In most cases this occurs when the topics chosen are unsuitable and should have been discouraged by the teacher. Most of the candidates explained how they were going to collect their data, but not all of them described the mathematical techniques they were going to use in their project. Candidates with clear statements of task and plan generally wrote more successful pieces of work.

B. The data collected was generally relevant to the stated task. On the whole it was well collected and well organized ready for analysis. Data quality was frequently more of a problem than data quantity. A few candidates did not include their raw data so it was difficult to see how this had been structured ready for analysis. Some candidates forgot to include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given making it impossible for the moderator to check whether or not the tables of data were accurate. A large number of candidates just downloaded data straight from the internet with little thought being given to how much of that information was really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way. It is also important to state the website in the bibliography.

C. Almost all candidates attempted simple mathematical processes and the chi-squared test was the most popular sophisticated process. Candidates and teachers need to remember that the processes also have to be relevant. Some projects contained many mathematical calculations, some of which were meaningless for the actual project. The importance of relevance must be stressed. When a scatter diagram indicates that there is no correlation between two variables then it is meaningless to go on to calculate the correlation coefficient and the line of best fit. Working out standard deviations without a meaningful discussion on what the results indicate is of no value.

   Many candidates relied entirely on computer generated results and no explanations of the techniques used appeared in their project. With some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. A

growing number of candidates and teachers do not realize that no more than 20% of the expected cells can have values less than 5 and none of them should have values less than 1 for the test to be valid. Also several candidates used raw data instead of frequencies.

D. Almost all the candidates were able to produce one conclusion or interpretation that was consistent with their analysis but often these were rather brief. In many cases the conclusions were just a one or two line statement. This does not produce a meaningful discussion of results. Teachers need to encourage candidates to ensure that their interpretations and/or conclusions are developed in a comprehensive way. Sometimes candidates were unable to discuss their interpretations/results through their own lack of understanding of the relevance of the process used.

E. Validity was generally attempted by most candidates although very few achieve level 2 for this criterion. The majority of the candidates were successfully able to comment on their data collection method or their interpretations and/or conclusions but few commented on the mathematical processes that they had used or suggested possible means to improve their project.

F. The overall presentation of the project work was good. The majority of the projects are now word processed, hence easier to read with tables and graphs clear to follow. In some projects questionnaires used for surveys were not included and in others the data was either missing, not set up for use or was relegated to an appendix. In most projects correct mathematical language was used. Many candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

G. The majority of the teachers appear to have awarded marks appropriately.

## Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Write clear and full comments on the 5/PJCS forms.
- Discuss a detailed plan of the project with the candidate to see if the task chosen has a mathematical future.
- Describe the sampling technique used to collect the sample.
- Encourage candidates to work on the evaluation area of their project in more depth.
- Encourage candidates to use a wider variety of mathematical techniques both simple and sophisticated.
- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.

- Emphasize the importance of showing, where appropriate, sample calculations in both simple mathematical processes and sophisticated techniques.
- Stress the importance of using appropriate notation and terminology.
- Stress the importance of documenting sources. Where does the data being analysed come from.
- Discuss in class the significance and limitations of specific techniques.
- Give candidates a second chance to correct errors.
- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Tell them to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.

# Standard level paper one

## Component grade boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 - 11 | 12 - 21 | 22 - 35 | 36 - 48 | 49 - 60 | 61 - 73 | 74 - 90 |

## General Comments

This paper proved to be accessible to most of the candidates. Time did not appear to be an issue for the majority of candidates. The comments on the G2 forms were encouraging, all judging the paper's level to be appropriate. There was a wide range of marks with the great majority of candidates from some schools scoring well. However, it appeared that other schools did not prepare their candidates properly for this examination as certain topics were omitted by all students. The treatment of accuracy and unit penalties showed an improvement over previous years but still too many candidates are not following instructions in this regard. Relatively few lost the financial penalty. The graphic display calculator (GDC) was not always used effectively, most notably in the statistical questions 6 and 11. The questions that posed the most problems were questions 6, 7, 13 and 15.

The Spanish G2s pointed out the error that occurred during translation in Q7 where in line 3 $L_2$ had been incorrectly written as $L_1$. All the Spanish scripts were looked at before the Grade Award meeting and those candidates who had been affected by the error were identified and compensated accordingly. It was estimated that about 10% of all Spanish candidates had been affected and their results were changed prior to the Grade Award meeting**.**

## The areas of the programme and examination which appeared difficult for candidates

The GDC was not used properly or to its full capability in answering some of the questions; too many were unable to obtain the correct answer to question 1(a) and the statistical functions were not utilised especially on question 6, which was perhaps the least well attempted question, and on question 11. It may well be that candidates are not aware that, following a reset of the TI84, the GDC must be adjusted to access the correlation coefficient.

The financial question caused problems for many as did the questions that included the use of parameters and some algebraic manipulation. Functions remain an area of weakness for many.

## The areas of the programme and examination in which candidates appeared well prepared

Working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect, however, where the GDC is used care must be taken to enter data and parameters correctly.

The questions on sets and basic statistics were well answered by the majority of the candidates. It was encouraging to note the improvement made in the treatment of sequences in questions 8 and 10.

# The strengths and weaknesses of candidates in the treatment of individual questions

**Question 1: Accuracy and standard index form**

(a) This was answered correctly by the majority of the candidates however some candidates entered the numbers incorrectly and arrived at the wrong answer.

(b) (c) Correction to decimal places was less well attempted than to significant figures.

(d) Most made a successful attempt to change their answer to part (a) into scientific notation. Some were penalised for not using their answer to (a)

**Question 2: Statistics – raw data**

Parts (a) and (b) were well done by the vast majority of candidates.

Part (c) caused problems to many – with (1) the mean of the two grades not being taken (2) the mean being calculated instead of the median.

Part (d) was successfully completed by those candidates who did the question by counting. Those who tried to use the probability laws were not successful.

Much of the question could have been checked by inputting the data into the GDC.

**Question 3: Statistics – cumulative frequency graph**

Candidates showed less facility in this question compared to question 2.

(a) was generally answered well. There were a number of inaccurate readings from the graph.

(b) Errors came from candidates who either used the $x$ coordinates for the quartiles or who wrote the quartiles as an interval, rather than subtracting these.

(c) was well attempted from the candidates' (a) and (b)

**Question 4: Logic**

(a) was generally answered well.

(b) lack of precision in language led to many errors.

**Question 5: Exchange Rate**

(a) This caused problems for many candidates. The form of the exchange rate proved difficult.

(b) Most candidates managed to answer this correctly.

(c) This part also proved problematic for many candidates.

**Question 6: Statistics – Histogram**

The class boundaries needed to be correctly identified to permit full credit to be given. Weight being a continuous variable and given to the nearest kg meant that the lowest class boundary was 39.5. Thereafter, the use of midpoints is standard.

(a) The endpoints of the bars caused problems for all but a very few candidates. Diagrams drawn without a ruler were also penalized.

(b) This was well attempted by the majority; it acted as a prompt for the following parts.

(c) (d) resulted in many incorrect answers; it was expected that the GDC would be used for these parts of the question, though a number calculated the mean by hand .

**Question 7: Intersection of Perpendicular Lines**

(a) The omission of the negative sign was a common fault.

(b) Most candidates managed to answer this correctly from their (a).

(c) This part proved challenging for the majority. Once again, the use of the GDC was expected.

**Question 8: Arithmetic Sequence and Series**

(a) Again, the omission of the negative sign was a too common fault.

(b) This was generally well attempted.

(c) The common misconception was confusion between $k$ and the value of the $k^{th}$ term. Close reading of this part was required from the candidates.

**Question 9: Venn Diagram**

This question was well attempted by the majority. The major error was the omission of the "6" in the candidates' calculations. Perhaps better positioning would have helped in this regard.

**Question 10: Compound Interest**

The use of the TVM solver, and consequent lack of working, was a source of concern; candidates are advised still to write down substituted formulas prior to using the TVM solver.

The use of 8% in the second part was a common error. The compounding period was again a discriminator for the candidature.

**Question 11: Correlation and Regression**

The level of accuracy required by the paper was often ignored in this question.

(a) Some candidates are unable to recover *r* from a reset calculator.

(c) Many candidates seem to be unaware when it is appropriate to use a regression line.

**Question 12: 3D Trigonometry**

This question was poorly answered by many of the candidates. Pythagoras was improperly applied and candidates were unable to identify right angled triangles.

**Question 13: Quadratic Function**

This question was poorly answered by all but the best candidates. The links between the parts were not made. The idea of the line of symmetry for the graph was seldom investigated. The “minimum value of the function” was often incorrectly given as a coordinate pair.

**Question 14: Mapping Diagram for Exponential Function**

(a) The concept of the zero index was not understood by the majority.

(b) This part was often left unanswered.

(c) Only the best candidates were successful in this part.

**Question 15: Sine Functions**

(a) (b) Candidates were either fully successful or not at all in determining the functions. Some were successful but did not give answers as “equations”.

(c) Those successful in (a) and (b) usually completed the question, but there were occasions where the answer was given (incorrectly) as a coordinate pair.

## Recommendations and guidance for the teaching of future candidates

The candidates need to gain more confidence in using their GDC. Further practice with both the numeric and graphical functions of the GDC would benefit all candidates.

It is further expected that the great majority of statistical calculations will require the use of the GDC. Candidates should be reminded that errors in inputting data cannot be addressed by markers since no working is shown – hence **great** care must be taken and the entries checked.

The entire syllabus will be tested over both papers (though not necessarily in each) and must be taught.

Teachers need to remind candidates to give answers to the accuracy required in a question, or to 3 significant figures otherwise. They must also be aware that a unit penalty will be awarded in all instances where units are required (though **not** for currencies). The financial penalty causes fewer problems, however,

All relevant working should be shown in each question so that follow through marks can be awarded when necessary. With the more frequent use of the financial application, it should be emphasised that the only working that can be shown is via the substituted formula.

# Standard level paper two

## Component Grade Boundaries

| **Grade:** | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| **Mark range:** | 0 - 11 | 12 - 22 | 23 - 34 | 35 - 47 | 48 - 59 | 60 - 72 | 73 - 90 |

## General Comments

The great majority of the candidates attempted all the questions and were thus able to gain easier marks on each of these. In each question many candidates were able to score the maximum, indicating that the questions were accessible to those who had covered the entire syllabus.

Very few students were penalized with both the Accuracy and the Unit Penalty for not giving answers correct to three significant figures and for not writing the unit in their answers respectively. Frequently premature rounding resulted in the final marks being lost. This occurred mainly in Question 1 and Question 2. Also a number of candidates lost the final mark in some “show that” questions for not writing down the unrounded answer.

Although the candidates did not appear to have had problem finishing the paper in the time allocated, the fact that there were two graphs to draw on the paper proved time consuming for some candidates. Also time was lost in working the questions out manually particularly in Question 1 where candidates tried to work out the terms for both progressions by hand. It can be said that the use of the GDC has improved. However, there are still some GDC tools (e. g. table mode for listing terms in a sequence and drawing graphs; equation solver) that candidates are not using when answering the questions. These tools may minimize the time spent and help increase the number of gained marks.

Where clear working was shown, follow-through and method marks could be awarded when the answers were incorrect.

## The areas of the programme and examination which appeared difficult for candidates

The following were found to be the most common areas of difficulty.

- Use of AP and GP formulae to find terms
- Volume of a prism

- Conditional probability
- Probabilities of events using “without replacement”
- Graphical method to solve an inequality
- Values of $x$ where $f'(x)$ is given
- Tangents to a curve: How to draw them?
- Equation of a tangent at a maximum point

## The areas of the programme and examination in which candidates appeared well prepared

The following topics were all well answered by a number of candidates:

- Sine and cosine rule
- Area of a quadrilateral
- Chi-squared test and use of GDC to find the $\chi^2_{calc}$.
- Simple probabilities
- Graph of exponential and linear functions in a given domain
- Graphical solution of unfamiliar equations
- Use of GDC to sketch the graph of a function
- Differentiation
- Meaning of the derivative

## The strengths and weaknesses of candidates in the treatment of individual questions

**Question 1: Arithmetic and Geometric sequences**

This question was well answered by the majority of the candidates. Most of the candidates were able to distinguish between the arithmetic and the geometric progression. A number of candidates worked out term by term by hand for which they needed more time than those that used the formulae to find the requested terms. Some of the students that found the terms the long way also lost a mark for premature rounding. It was pleasing to see how the last part of the question was answered using different methods. Those candidates that worked throughout the question using AP and GP formulae used either the solver or a graph to find the solution of the inequality. Those candidates that worked throughout the question in the long way also managed to compare the terms and find the correct year.

**Question 2: Trigonometry-Volume of three dimensional shapes**

It could have been written that the diagram was representing the plan of the sandbox. However, examiner's comments did not find this lack of information an obstacle for the candidates.

Many students lost the unit penalty in this question and others also lost the accuracy penalty. Overall the lengths of AC and AB were well done. Sine rule and cosine rule were in general well used. To find the length of AB many students used correctly right- angled trigonometry. The area of the sandbox was in general well done though some students did not gain the final mark due to premature rounding or for not showing the unrounded answer. The volume of the prism was poorly answered by the majority of the students. Most of the students did not use the correct formula. Very few candidates noticed that the value 40 was given in cm. It was good to see very few students losing marks for having their GDC setting in radians.

**Question 3: Test for independence – Probability**

The first part of the question was relatively well done. The null hypothesis and the degrees of freedom were well answered by the majority of the students. In the show that question some students used the GDC to find the expected values table and highlighted the correct value 22.05. This procedure gained no mark; the expected value formula was expected to be used here. Also those who did use the formula were expected to show the unrounded value 22.05 to gain full marks in this part question. Many lost the answer mark for not doing so. GDC was used by most of the students to find the chi-squared test though some students attempted to find this value by hand which made them waste time. Correct values were compared when deciding whether to accept or not the null hypothesis and follow through marks were awarded from their degrees of freedom and chi-squared test when incorrect.

The second part was not as successful as the first one. Simple probability was well answered. Not all the students changed the denominator to 45 for the second probability showing their weaknesses in conditional probability. It would have been useful for the students to use a tree diagram to help them solve the last part of this question but very few did so. Some of those students that reached the last part of the question forgot to add one of the three terms. Very few used the probability of the complement.

**Question 4: Graphs of exponential and linear functions**

Good marks were gained in this question, mainly from parts (a) to (d). Very few students answered the show that question using a backwards process. These students did not gain full marks. Labelled and neat exponential graphs were drawn. Marks were lost sometimes for starting the curve at point (1, 66.9) instead of at (0, 90). Also there were students using a ruler to help them joined up the points for which they lost one mark as the graph must be smooth. Candidates managed to find the time at which the temperature was 56. However, those students that gave their answer as a coordinate pair lost the answer mark. Some students justified the behaviour of the curve by mentioning the asymptote but the big majority said that the room temperature was 20 and it was awarded full marks. Most of the students drew the straight line correct and in the given domain. Those students that drew the line in a separate set of axes could not answer the last part of the question. This part question proved to be

difficult for many students and so worked as a discriminating one. One of the most common errors seen in part (b)(iii) and (e)(i) was to give the answer as a point instead of giving just the first coordinate of that point.

**Question 5: Calculus**

Many candidates managed to gain good marks in this question as they were able to answer the first three parts of the question. Good sketches were drawn with the required information shown on them. Very few candidates did not recognise the notation $\frac{dy}{dx}$ but they showed that they knew how to differentiate as in (d)(i) they found the derivative to show that the gradient of $L_1$ was 12. Candidates found it difficult to find the other $x$ for which the derivative was 12. However, some could draw both tangents without having found this value of $x$. In general, tangents were not well drawn. The last part question did act as a discriminating question. However, those candidates that had the function drawn either in their GDC or on paper recognised that at $x=-2$ there was a maximum and so wrote down the correct equation of the tangent at that point.

## Recommendations and guidance for the teaching of future candidates

- Use of the AP and GP formulae to find terms in order to avoid listing and make better use of the exam time
- Teach how to draw a tangent to a curve at a given point
- Show that questions: how to answer them?
- Tell your students to round the answers in the very last step of the working and not in intermediate steps. Avoid premature rounding.
- Remind your students to give the answers to the accuracy required in a question, or to 3 significant figures otherwise.
- Make sure that the students recognise the different notations for the same concept (i.e. $\frac{dy}{dx}$ and $f'(x)$)
- Tell your students that most of the calculator commands are not mathematical notation and are not acceptable as working in the exam.
- Guide the candidates on time management
- Be familiar with the information booklet and know which formulae are given there by using it in a daily basis in class.

- Tell them to show all relevant working in each question so that follow through marks can be awarded when necessary.