

## MATHEMATICAL STUDIES

### Overall grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 15	16 - 28	29 - 40	41 - 55	56 - 69	70 - 82	83 - 100

### Standard level internal assessment

#### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The vast majority of the projects this session were appropriate, if lacking sometimes in originality and content. In some projects the tasks chosen were too narrow but in most cases the candidates did have sufficient scope to demonstrate their mathematical ability. It was a pleasure to read some projects based on Graph Theory. There were also some projects involving Mathematical Modelling and others with Calculus and Trigonometry combined and it was really satisfying to read some of these projects. However, projects with a statistical basis still predominate.

### The range and suitability of the work submitted

Most of the projects were well presented with few being hand written. There were a number of very short projects. The internal assessment is intended to be a substantial piece of work and three or four pages of simple mathematics will not score highly in a number of criteria areas. A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the calculations.

There was a significant increase in the number of candidates using the chi-squared test and linear regression. A major concern is the number of candidates and teachers who do not realize that not more than 20% of the expected cells can have a number less than 5 and that no expected cells can have a number less than 1 for the test to be valid. Also there is no point in finding a correlation coefficient or regression line if a scatter diagram has shown that there is no correlation.

More candidates are now using their GDC to do the mathematics for them and often they forget to write down the formula they are using and mention why a particular procedure is

being used. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

When using the internet the candidate must remember to include the web address in their bibliography.

More teachers are now writing useful and pertinent comments on the cover form. This is useful for the moderation process.

## Candidate performance against each criterion

### Criterion A

Most of the topics chosen were appropriate for a Mathematical Studies project. The majority of the projects had a title. There are still some candidates who find it difficult to explain in a clear and concise way their statement of task. In most cases this occurs when the topics chosen were quite difficult to understand. When describing the plan, many candidates explain what they are going to do to collect their data, but only some of them describe the mathematical techniques they are going to use in their project. In many projects the method used to generate the sample was not specified. The plan lacked details such as who would be sampled and the rationale for that selection. The source of the data was unclear in many projects. Candidates with clear statements of task and plan tended to be able to extract more depth from their projects because they knew what they were looking for.

### Criterion B

The data collected was generally of sufficient quantity but was not always focused on the task. It was easier to find projects where the data could be considered enough in quantity but not in quality. A few candidates did not include raw data within their project or as an appendix, nor did some include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given. It is very difficult for the moderator to check accuracy in cases like this. Also, if a survey or questionnaire is handed out “at random” to a number of people then the candidate should explain what this “at random” means. A large number of candidates simply “dump” tables and charts straight from the internet into their project, with little thought being given to how much of that information is really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way.

### Criterion C

Most candidates used basic mathematical techniques for analysis, many relying entirely on computer generated results. Many of these candidates omitted explanations and clarifications of these techniques and were not selective about using the particular results that were

relevant for their investigations. Some candidates are applying sophisticated techniques in their analysis and are omitting the simple mathematics and/or the use of graphs to analyze their information. With some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. Why have several chi-squared tests in one project? Why find the equation of the regression line when it is clear from the graph that there is no linear relationship? Why find the equation of the regression line and then not use it? Also, the mathematics needs to be done in a meaningful manner. Some projects contained many mathematical calculations, some of which were not relevant for the actual project. The teachers differed in their interpretation of what constituted “sophisticated” mathematical techniques and this was an area that often required moderation.

#### **Criterion D**

Almost all the candidates were able to produce conclusions or interpretations that were consistent with their analysis but sometimes these were rather brief. In a high number of cases the conclusions were obvious and not very thorough.

#### **Criterion E**

More candidates than in previous sessions commented on validity. Usually this was more to do with the data collection than anything else. A few commented on the mathematical processes that they had used. Of those who did, few reached the level of thoroughness required for a high level of achievement.

#### **Criterion F**

Although in a few cases questionnaires used for surveys were sometimes not included in the project and in others it was difficult to follow the process because important data had not been set up for use or had been relegated to an appendix, on the whole, projects were easy to read and well structured. In most projects correct mathematical language was found and the tendency to repeat calculations and techniques on different sets of data and coming up with pages and pages of repetitive and often needless information has diminished to a certain extent. Many candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

#### **Criterion G**

The majority of the teachers appear to have awarded marks appropriately.

### **Recommendations for the teaching of future candidates**

Teachers can help their candidates in many ways:

- Encourage candidates to work on the evaluation area of their project in more depth.

- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.
- Emphasize the importance of showing sample calculations in both simple mathematical processes and sophisticated techniques and to present those calculations, regardless of the use of technology.
- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Tell them to state clearly their objectives and to comment on them once the project has been completed.
- Try to avoid repeating the same mathematical process several times.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Encourage them to think up their own task and explain the plan thoroughly.
- Advise the candidates to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the **original** work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.

## Standard level paper one

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 27	28 - 38	39 - 50	51 - 63	64 - 75	76 - 90

### General comments

This paper proved to be accessible to most of the candidates. Time did not appear to be an issue with most candidates attempting all the questions. The comments on the G2 forms were encouraging. There was a wide range of marks with candidates from particular schools nearly all scoring well. It appeared as if certain schools had not prepared their candidates properly for this examination as certain topics were omitted by all their students. Nearly all the candidates lost an accuracy penalty and a unit penalty but few lost the financial penalty in this paper. There was an improvement in the use of the graphic calculator (GDC) in this paper compared to previous years. The questions that posed the most problems were questions 12, 13 and 15.

It was noted that some schools are still using the information booklet with the **wrong formula** for percentage error. The IB coordinator must make sure that the correct information booklet is downloaded from IBNET.

### The areas of the programme and examination that appeared difficult for the candidates

Many candidates lost marks due to lack of accuracy and missing out units. Few lost a mark due to the financial penalty.

A number of candidates did not use the GDC to its full capability in answering some of the questions on this paper. Time was lost in working the questions out manually. This is especially true in Question 14 where some candidates tried to work out the equation of the regression line by hand.

Question 12 with domain, range and vertical asymptote was the question that proved to be difficult for many of the candidates. A significant number of candidates had their calculator in radian mode and had difficulty with question 9. Solving a linear equation was poorly answered and so was finding the perpendicular bisector of a line joining two points. Question 15 on calculus was also difficult for many candidates.

Many candidates also had problems finding the total number of students from their Venn diagram.

## The areas of the programme and examination in which candidates appeared well prepared

Good working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect.

Sets, arithmetic sequences, statistics, matching given graphs to given functions, simple and compound interest and logic were well answered by many of the candidates.

## The strengths and weaknesses of the candidates in the treatment of individual questions

### Question 1 Accuracy and percentage error

- (a) This was answered correctly by the majority of the candidates however some candidates entered the numbers without using brackets and arrived at the wrong answer.
- (b) Most made a successful attempt to change their answer to part (a) into scientific notation.
- (c) (i) Many candidates managed to find the answer but then lost the mark by not adding the units.  
  
(ii) Several candidates are still having a problem finding the percentage error. The formula is given in their information booklet and they should have had practice using all the formulae that they are given. There are some schools that are still using the incorrect formula sheet for percentage error.

### Question 2 Set notation

Parts (a) and (b) were well done although some candidates added 1 as a multiple of 3.

Part (c) was reasonably well attempted although some candidates found the intersection instead of the union.

Part (d) was successfully completed by those candidates who managed to find the complement of B correctly. If they had not shown the set containing the complement of B in the working they could not be awarded the method mark.

### Question 3 Arithmetic sequence

This question was generally answered well. Most of the candidates had a good understanding of how to use the formulae for an arithmetic sequence.

**Question 4 Statistics and Box and Whisker diagram**

- (a) Most of the candidates knew how to find the mean birth weight but made arithmetical errors. Missing out the decimal point was a more serious error as it resulted in a mean birth weight of 31 kg which is rather unrealistic for a baby!
- (b) The median and upper quartile were correctly calculated on the whole.
- (c) The box and whisker diagram was well done although some candidates plotted their mean value instead of the median.

**Question 5 Matching given graphs to given functions**

Nearly all the candidates scored 6 marks for this question. Without any working shown it was difficult to say where the errors might have arisen from the few candidates who did not score full marks. However, it was obvious that the candidates were using their GDC's to graph the functions.

**Question 6 Simple and Compound Interest**

This question was also well done. However many candidates only gave the interest instead of the total investment. Some also lost a mark by failing to give the answer to part (b) to 2 decimal places.

**Question 7 Logic**

- (a) This caused problems for many candidates. They seem to expect to include the implication symbol somewhere.
- (b) Most candidates managed to write this correctly.
- (c) Not all candidates could complete the truth table correctly. Many managed the first column but then made mistakes in the last column.

**Question 8 Venn Diagram**

Venn diagrams continue to be a problem area. Quite a good number of candidates managed to fill in the information on the Venn diagram accurately. However, finding the correct value for  $x$  and calculating the number of students in the school posed a big problem for many candidates.

**Question 9 Non Right-angled triangle**

The candidates who used the cosine and sine rules for this question were successful on the whole. Some had their calculators in radian mode (and hence the second answer for the angle was unrealistic) but this was less frequent than in previous sessions. Those candidates

who used right-angled trigonometry scored no marks. Many candidates lost an accuracy penalty in this question.

### Question 10 Algebra

- (a) Many candidates forgot that a minus times a minus gives a plus and so did not solve the equation correctly.
- (b) A good attempt was made at factorising the function although  $x(x + 2) - 3$  was seen frequently too.
- (c) Few candidates realised that they had to use their GDC to find this answer and hence there were few correct answers. Some did not read the question correctly and solved part (b) to find the positive solution of the expression they had factorised.

### Question 11 Sketching a Sine curve

- (a) Some candidates sketched the graph accurately from their GDC. Others were not careful with turning points and many copied the graph completely wrongly. Some did not have their calculators in degree mode and trig graph mode and graphed straight lines.
- (b) and (c) the marks here were given for both the correct answer and as follow through from the candidates graph. Many lost marks for the amplitude by giving the answer as the maximum value minus the minimum value.

### Question 12 Mapping diagram, domain, range and asymptote

This question was poorly answered by many of the candidates. Some could answer part (a), finding values, but most could not give the domain and range of the function in (b). It was clear that many did not understand the question and this was an area of the syllabus that had not been fully taught. Giving the equation of the vertical asymptote was poorly answered also.

### Question 13 Coordinate geometry

- (a) It was surprising how many errors were made in finding the values for  $s$  and  $t$
- (b) The candidates had difficulty in finding the equation of a straight line. They could find the gradient of the line AB and a number could give the gradient of the perpendicular line but most did not substitute the correct midpoint to find the equation of the line.

### Question 14 Bivariate statistics

This question expected the candidates to use their GDC to find the equation of the regression line and to find the correlation coefficient and this was stated in the question parts. However,



there were a number of candidates from specific schools who had not been taught to do this and they tried to find the equation from the formula in the information booklet. This would have wasted time and they did not manage to find the correct answer. This was also the case for the correlation coefficient. However, most candidates who found a correlation coefficient managed to comment on it correctly.

Many candidates did not give a whole number answer for the number of objects in part (b).

### Question 15 Calculus

- (a) Many candidates gave up at this point. Those who attempted the derivative did so with varying success. Many could not differentiate a term with a negative index.
- (b) In part (b) most substituted the -1 into the original function rather than the differentiated one. They did not realize they had to put the differentiated function equal to zero.

## Recommendations and guidance for the teaching of future candidates

- Candidates need as much practice as possible in answering questions written in different styles.
- The whole syllabus must be taught and teachers need to be aware of any changes to the syllabus that have occurred.
- Teachers need to remind candidates to give answers to the accuracy required in a question, or to 3 significant figures otherwise. They must also be aware that a unit penalty can now be awarded and also a financial penalty. A number of candidates lost 3 marks, some lost 2 marks and most lost 1 mark on the paper.
- All relevant working should be shown in each question so that follow through marks can be awarded when necessary.
- The candidates need to gain more confidence in using their GDC. Many can now sketch graphs accurately but cannot find mean, standard deviation, correlation coefficient, regression line or chi-squared statistic.

## Standard level paper two

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 11	12 - 23	24 - 35	36 - 48	49 - 61	62 - 74	75 - 90

### General comments

The paper proved straightforward with most candidates managing to attempt all the questions. There was a marked difference in the quality of work from different regions. In general, candidates from Australian schools were familiar with all aspects of the course content and scored well. This was not true for candidates from schools in Africa or South America, most of whom appeared to be insufficiently prepared and unable to attempt much of the paper including the tree diagram or the basic trigonometry question.

Candidates are requested to start each question on a new sheet, but many did not.

Almost every candidate lost an Accuracy Penalty, usually from writing an answer to 4 significant figures instead of 3. Many candidates also lost a Units Penalty particularly in question 3.

Premature rounding during questions again resulted in final marks being lost.

### The areas of the programme and examination that appeared difficult for the candidates

It is not easy to say which areas most candidates found difficult as it appeared to vary with each centre. While some candidates found the trigonometry question difficult, others found it easy, and so on with most of the questions. The following were found to be the most common areas of difficulty.

- Using GDC properly to find mean and standard deviation, especially from grouped frequencies.
- Using GDC to find points of intersection
- Equation of asymptote
- Knowing to differentiate to find the value of the gradient at a point
- Finding coordinates where the tangent is parallel and perpendicular to a given line
- Conditional probability

## The areas of the programme and examination in which candidates appeared well prepared

The best area was the financial section, where almost everyone obtained very good marks. Finding the value of chi-squared by GDC was also well done by a large number of candidates.

## The strengths and weaknesses of the candidates in the treatment of individual questions

### Question 1 Probability / Graph sketching / Calculus

This should have been an easy first question but, even so, there were some candidates who were unable to fill in the tree diagram correctly let alone evaluate any probabilities. The majority of candidates were confident with answering parts (a), (b) and (c) but the conditional probability question was not well answered with few candidates managing to recognise that it was a conditional type.

The curve sketching and straight line were well drawn but not all candidates indicated the intersection points with the axes. In finding the line / curve intersection some candidates did not use the intersection function on the GDC. Few candidates managed the last part. Many just chose two sets of coordinates and used the gradient formula.

### Question 2 Trigonometry

The volume of the pyramid and the weight were well done. Many candidates lost their unit penalty here. They had trouble showing that the sloping edge was 3.6 cm. The angle BVC was done well but not the total surface area. They knew that they needed four sides and the base, but finding the area of the triangle proved difficult for the less able candidates.

### Question 3 Functions / exchange rates

Many candidates who had not lost a UP in question 2 lost one here. Parts (a), (c) and (d) were reasonably well tackled. Almost everybody had difficulty with the equation of the horizontal asymptote, a common answer being  $y = 20$ . Most of the candidates realised that 30 seconds was 0.5 minutes and calculated part (e) correctly. Part (f), solving an exponential equation, was a good discriminator. Trial and error was expected but many students did not think of doing this.

The financial part was the best done question in the paper and a large majority of candidates gained full marks here.

**Question 4     Statistics**

Many candidates who had survived the previous two unit penalties, fell here with omission of units for the mean and standard deviation. The modal group was answered well. Part (b), finding the mean and standard deviation by GDC, was answered very poorly. Most did put the midpoints in one list and the frequencies in a second list but then either used the 2-Var stats button or 1-var stats button but only named L1 instead of L1, L2. Candidates who showed midpoints in their working did at least score a method mark.

The chi-squared question was answered well by the majority of candidates and almost all found the chi-squared statistic correctly by GDC, though many could not look up the correct critical value.

**Question 5     Coordinate geometry / calculus**

Parts (a) and (b) were very well done. After that, only the stronger candidates were able to cope. The equation of the tangent at the point with coordinates (2, 6) was badly done but some candidates managed to find the equation of the tangent line from their GDC. The equation of the axis of symmetry was reasonably well done although many just wrote down 1.5 instead of  $x = 1.5$ .

Some forgot to write down that the gradient at the vertex was 0.

**Recommendations and guidance for the teaching of future candidates**

- More practice using their GDC
- Cover all topics on the syllabus
- Guide the candidates on time management
- Be careful to give answers to the correct level of accuracy
- Be aware of the consequences of premature rounding
- Put in all units
- Show all working
- Be familiar with the information booklet and know which formulae are given there.