# MATHS STUDIES SL

## **Overall grade boundaries**

## **Standard level**

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 29	30 - 40	41 - 54	55 - 68	69 - 81	82 - 100

# Standard level internal assessment

### **Component grade boundaries**

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

## The range and suitability of the work submitted

The projects from almost all candidates were based on statistical analysis of primary or secondary data with chi-squared and regression prominent. There were just a few projects based on other topics including some algebraic modelling in a few projects which were largely successful. It would be good to see candidates being encouraged to be more adventurous in this area.

There was a noticeable improvement in the candidates' response to criteria 1 and 2 in projects seen this session, compared to the corresponding session last year. This is perhaps an indicator that the new criteria are clearer for teachers and students. The tasks were clearly stated in most cases and the plans generally included a description of both the method and mathematical processes being applied. The detail given in the plans was pleasing.

The individual tasks chosen were appropriate and offered the opportunity for candidates to achieve the assessment objectives. The better projects focused on a distinct objective with a clear hypothesis or direction. Candidates who used a 'broad' survey approach did not fare well.

The mathematical content was mostly sufficient with only a few projects being classed as too limited in this area. Invariably, these projects included large amounts of information downloaded from the Internet rather than analysis developed by the candidate.

Almost all candidates attempted to use more sophisticated mathematical processes, again an improvement from previous sessions. Correlation/regression and the chi-squared distribution were the most popular techniques applied which fits well with the course. There was an occasional sighting of the student 't' test, Spearman's rank order correlation and trigonometric and exponential models.

One disappointing aspect of the mathematical analysis was the number of projects which included incorrect chi-squared tests. They used raw data instead of frequencies and often had many values < 5, which reduces the reliability of the test. There were also many tests which used values less than 1, including zero, which makes the test irrelevant.

The written analysis of results was mostly satisfactory. Only a minority of candidates were able to gain both marks for validity with the usual failing being the lack of validation of the mathematical

processes used. What was pleasing here was that most candidates did write about the validity of their results to score one mark for this criterion.

The overall structure and presentation of the work was very good and again, an improvement from last year. Candidates generally acknowledged their sources and a greater proportion of projects included their raw data which assisted in determining the accuracy of their results.

# Candidate performance against each criterion

#### A: Introduction

As mentioned above, there was a significant improvement noted for this criterion. Most candidates gained both marks with a title given, tasks being clearly stated and a plan being described in good detail. Most projects were well-focused with a limited and well defined scope. There were some which provided a lot of detail but their plans were too broadly defined to provide a sound basis for analysis.

Better projects are specific in their expectations and clearly identify the practical and mathematical processes that will be used.

#### **B:** Information/Measurement

The collection of data and/or measurements was quite satisfactory and the material was generally well organised for analysis. The construction and implementation of surveys to collect primary data was not always done well, mostly a result of asking too many questions.

The quantity of data was impressive in many cases, whether gathered from surveys or the internet. The quality was not always there, but at least the candidates had sufficient material from which to draw sound conclusions.

The better projects included some discussion of the data collection process, Data does not appear from nowhere! The aim is to use random data to eliminate bias. Discussing the method of collection can gain a tick for validity, irrespective of whether the data is randomly gathered or not.

Quantity of data is a difficult facet to comment on - it does depend on the nature of the task undertaken. In many cases, the task itself can be a limiting factor in obtaining quality data.

#### **C:** Mathematical processes

As mentioned above, the use of these techniques was often limited being no more than an extension of a classroom exercise.

A larger proportion of candidates than in the past were able to gain the maximum score under the new criterion. They used more than one 'sophisticated' (relevant) process and did so accurately. The question of relevance is important here. Many students use correlation and regression or chi-squared, as their analytical procedures and often these are not appropriate for the context or data being analysed. Calculating the equation of regression when the correlation coefficient is negligible is a useless exercise. Similarly, using raw data and not frequencies for a chi-squared test is inappropriate.

Standard deviation continues to disappoint as a statistic which is often calculated but not used or commented on and when it is, the commentary is often meaningless. "Data having a wide dispersion" is an irrelevant statement on its own.

Many projects lack an explanation of mathematical techniques, merely the calculation and interpretation of results. This seems to be improving but is still very evident.

#### **D:** Interpretation of results

This was done well by the majority of candidates. Many projects demonstrated a good understanding of the mathematics being applied and the interpretation of their results were consistent with their analysis. The better candidates provided a thorough and detailed discussion of their results.

The defining difference between scores was the ability of the candidates to produce a comprehensive discussion. Sometimes it is what the results do not show that needs comment. For example, the non-causation element of correlation.

#### E: Validity

The course now has this as a separate criterion and the improvement in candidates' attempts to satisfy this criterion was very noticeable. The majority of projects did include some discussion with regard to the validity of their results. Many tried to satisfy both elements; validity of process and results and, as a consequence, a significant number of projects were awarded both marks.

The general comment of "if I used more data …" does not gain a mark in its own right. The marker simply thinks, "So why didn't you gather some more data?". Any discussion along this line must include some detail about why more data was not available and, if the reason(s) given is (are) valid, then the mark could be obtained.

#### F: Structure and communication

The structure of most of the projects was sound. The students generally used appropriate mathematical language and presented the material correctly and in a systematic manner.

The better projects include their discussion through the body of the work. Projects which have page after page of mathematics followed by a 2000 word summary do not impress.

The overall presentation of the projects was good.

#### G: Commitment

The teacher awards the mark as he/she determines. There appears to be a wide range of justifications applied though these are usually sensible.

# **Recommendations for the teaching of future candidates**

There is little difference in this list compared to previous sessions. The same limitations and errors are always evident.

The task should aim to be specific rather than broad. Surveys need to have a mathematical rather than a social focus. The quantity and quality of data is a primary determinant of the final score for any project.

Projects need to provide some depth in the mathematical techniques applied. Projects based on simple methods make it extremely difficult for students to present any meaningful interpretation. Applications which simply mirror class exercises limit the opportunity for maximum scores.

Comparative studies often produce better results than single focus studies. Projects based on data from Economic, Geographical and Scientific areas often provide greater scope for meaningful interpretation than localised surveys.

Students should be encouraged to extend the mathematical processes applied. There is often an opportunity to incorporate algebraic models in many statistical situations. Regression, other than linear, is not too great a step for Mathematical Studies students to consider.

Greater emphasis needs to be placed on the concept of 'validity'. There needs to be an awareness of the appropriateness of certain techniques and of a project's limitations. Using the mathematical process correctly is imperative. Some research into the use of the procedures can have a significant positive effect.

Teachers and students need to be reminded that the project represents 20% of the final mark. It is a major piece of work that should demonstrate a commitment of time and effort by the candidate. Appropriate classroom time should be incorporated into the teaching programme.

# Standard level paper one

## **Component grade boundaries**

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 37	38 - 49	50 - 61	62 - 73	74 - 90

## **General comments**

The candidates, in general, answered this paper quite well. There was a wide range of questions offering a good coverage of the course. The individual questions appeared to be accessible to the candidates and they had enough time to make a reasonable attempt at all the questions. The overall standard of the paper was good.

There was an error in the text of question 9, part (b), in the English version of the paper only. An erratum slip was sent to schools before the examinations took place, but not all candidates were made aware of the mistake before they sat the examination. In light of this problem, it was decided to award all candidates full marks for the question part that was affected by the error. This applied to candidates who sat the Spanish paper also, to ensure that no candidate was disadvantaged by this error.

# The areas of the programme and examination that appeared difficult for the candidates

Most candidates did not use the GDC to its full capability in answering many of the questions on this paper. In many cases, time was lost in working the questions out manually. In a number of cases, those topics that were new to the syllabus were not well answered. In most cases, it appeared that the candidates had partial knowledge of the topics from which these questions were drawn and certainly very little practice with a range of different questions from these topics.

The *exclusive or* in logic was not well known by many candidates.

A number of students showed a poor performance in interest compounded quarterly.

The relationship between the period of the sine function and the coefficient b proved to be difficult for the majority of the candidates.

Calculating the expected frequency and the meaning of the p-value in the chi- squared test seemed not to be clear for the students.

Interpreting a graph where two functions are involved also caused trouble to some candidates.

#### SUBJECT REPORTS - NOVEMBER 2006

Sketching curves of unknown functions properly and putting in asymptotes was poorly done.

The majority of the students was not able to describe an interval using the appropriate inequalities.

# The areas of the programme and examination in which candidates appeared well prepared

Many candidates were able to score maximum marks in a range of questions, confirming that the questions were accessible. Good and clear working was shown by the majority of candidates so that follow-through marks and methods marks could be awarded when parts of questions were answered incorrectly. Unfortunately, a number of candidates still show minimal amount of mathematical working which prevents them from maximizing their marks.

The following topics were in general well answered by the candidates:

- accuracy,
- sequences, statistics (box and whisker plots),
- calculus, measurement,
- finance,
- exponential functions,
- frequency histograms.

# The strengths and weaknesses of the candidates in the treatment of individual questions

#### Question 1 Accuracy

Most candidates could answer this question and many scored full marks, however, a significant number gave the first answer as the full calculator display instead of giving the answer correct to 3 significant figures. This resulted in the loss of one mark because of accuracy. Many students forgot the brackets when using the GDC. Also a number of students seemed not to be clear about the difference between 3 significant figures and 3 decimal places in the second part. Follow-through marking was used throughout this question.

#### Question 2 Logic

A significant number of candidates omitted the "but not both" in (a) showing that they were not familiar with the exclusive "or". In (b)(ii) the inverse was often given instead of the contrapositive.

#### **Question 3** Simple and compound interest

The simple interest was well done except for some candidates who gave the total amount instead of the interest.

Not many candidates managed to find the compound interest correctly. Many candidates calculated this as annual compounds while others correctly used 24 periods but left the interest as nominal. It seemed that not many students knew how to use the TVM to check their calculations. Some confused quarterly with 3 periods (because of 3 months).

#### Question 4 Geometric sequences and series

This was one of the most successful questions in the paper. Unfortunately in part (c) they were able to substitute into the formula to calculate the sum of the first twelve terms but some did not manage to

get the correct answer. Often this occurred due to bad use of the calculator. Many candidates were penalised in this question because of accuracy as they gave their answer as just 16 instead of 16.0 correct to 3 s.f.

#### Question 5 Statistics

This question was well answered with the majority of the candidates scoring 5 or 6 marks.

The most common answer in (a) was t = 30.4 rather than the correct answer of t = 4. The markscheme allowed the candidates to be awarded one mark for this. The box plot was well done although there were candidates who were not able to use correctly the stated scale. Some examiners reported that it was clear that some centres had not taught this topic as their candidates left the question blank.

#### **Question 6** Differentiation

Most of the candidates could expand the expression though there were some who factorised it. A significant number of candidates could not differentiate the expression correctly. To find the gradient of the curve at -1 many substituted into the function rather than into the derivative. This showed a lack of understanding of the connection between gradient of the tangent and derivative. Again in this question part the GDC could have been used as a tool to confirm or find the answer.

The tangent of the angle in (d) was well answered by very few candidates. Many wrote down the measure of angle  $\theta$  instead of the value of tan  $\theta$ .

#### **Question 7** Trigonometric graph

Most candidates answered parts (a) and (b) correctly. Many candidates interpreted part (c) as length of time thus answered "6 hours". Many of those who interpreted the question correctly unfortunately did not use appropriate notation, which in this case was the inequality  $6 \le t \le 12$ . Instead they wrote down the answer "from 6 to 12".

Part (d) was poorly done. Very few students found the correct answer. Connection between period and the coefficient b was not even seen in the working. There were many good attempts to find b by setting out an equation using a point from the curve. They got lost while trying to solve this equation by hand. These candidates were awarded at least one method mark for showing their working.

#### Question 8 Measurement

This question was well answered with many receiving full marks for part (a). Some candidates forgot to set their GDC's to degrees and lost the final mark.

Part (b) was also well done, however, quite a number of students multiplied by 1.8 instead of dividing or did not take into account that the answer was asked to be given in minutes. There was good working shown in this question allowing follow-through marks to be awarded.

#### **Question 9 Quadratic function**

Many candidates had no idea how to do this. It seemed that they had not been taught it. There were many attempts to sketch the curve but poorly done. They did not realize that the question was asking about the symmetry of the curve. The most common answer to part (a) was (-3, 0) however the question was asking just for the x-coordinate, -3.

To compensate for the typographical error all candidates were awarded 4 marks for part (b) irrespective of their performance so the marks available for Question 9 varied from 4 to 6. The effect of this decision at each grade boundary level was considered carefully when finalising grade boundaries.

#### Question 10 Chi-squared test

The null hypothesis was correctly stated in the majority of the scripts and also the degrees of freedom. Not many candidates knew how to calculate the expected values. Hardly any candidates knew how to use the *p*-value to check whether to accept or reject the null hypothesis. Most candidates compared it to the critical value instead of comparing the *p*-value to the level of significance.

#### Question 11 Using the GDC with unknown curves

This question was designed specifically to be solved using the GDC. Part (a) was reasonably well done due to the increasing familiarity with GDC's, however, many lost marks because of accuracy and some because they traced the intersection points instead of using intersection.

Part (b) was poorly answered by many candidates who were not able to write the correct set of values for *x*.

#### **Question 12** Sketching the graph and finding the equation of the asymptotes

This was by far the lowest scoring question in the paper. Most candidates had no idea about asymptotes. The sketches were poorly done with very few candidates correctly drawing the left branch through the origin and many not bothering about the scale.

Even fewer candidates were able to write down in part (b) the equations of the horizontal and vertical asymptote.

#### Question 13 Currency exchange

This question was well answered with many candidates receiving full marks. They were able to convert one currency to another and could also understand commission. In part (b) some found 2.5 euros and gave this as their answer or gave the answer as a percentage (1.23%) rather than in pounds. These answers received partial marks.

#### **Question 14** Exponential decrease

Many candidates scored full marks on this question. Part (a) (i) and (ii) were very well done. Part (b) was also answered quite well though many candidates wrote down the correct equation but were not able to solve it. Some tried using logarithms but not many of these could reach the correct answer. The GDC solver could have been used to find the solution. The weaker candidates used 7999 instead of 8000 in the equation.

#### Question 15 Histograms: means and standard deviation

This question was generally well done. Many candidates scored full marks. It was difficult to see what errors were being made as the candidates were asked to fill in a table.

## Recommendations and guidance for the teaching of future candidates

This was the first November session in which graphical display calculators were compulsory and it was evident that a number of candidates were not able to use the calculator correctly. Students need to use the GDC continually during the 2 years of study to gain familiarity and confidence.

The whole course must be covered and teachers need to look carefully to be aware of the changes to the syllabus that have occurred.

It was clear this session that certain questions that asked for knowledge in a different style proved to be difficult to a number of candidates. Candidates need to be exposed to a wide variety of problems within each topic to ensure that they can establish appropriate problem-solving skills. This could be done by giving candidates practice with past IB papers.

Teachers need to remind candidates to give answers to the accuracy required in a question, or correct to 3 significant figures otherwise.

Of all the topics in the syllabus, the one which seemed weakest in paper 1 this session was "use of the GDC to sketch and analyse some simple, unfamiliar functions". Much more practice is needed in this area.

Teachers are also advised their students need to indicate clearly in their working which question part are they working in. If this is the case, examiners can follow through candidates' working when the answer is not correct and follow-through marks are easier to award.

# Standard level paper two

## **Component grade boundaries**

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 35	36 - 48	49 - 60	61 - 73	74 - 90

## **General comments**

This paper differentiated the candidates, as the marks ranged from full marks to very few marks on the paper. Each candidate appeared to have demonstrated his or her knowledge.

Many candidates appeared to be well prepared for this examination. Time did not seem to be a factor, with the majority of candidates attempting all the questions on the paper. There were some scripts that had parts of questions not attempted, mostly towards the end of the questions where the level of difficulty was higher.

This was the first November session in which the graphical calculator was compulsory and it was evident that a number of candidates had not been trained in using the calculator properly. A significant number tried to answer questions without using the capabilities of the calculator and lost time. If a question said 'write down' and was only worth one mark then that should have indicated to the candidate that the calculator should be used rather than trying to work out an answer using a page of working.

Although there were many neat scripts there were a number that were difficult to read with some still using pencil. These answers were very difficult to read in artificial light. The instructions on page two of the examination paper indicated that each question should be started on a new page. Some candidates had not been well prepared for this and there were scripts with more than one question on a page. Some did not have access to graph paper for accurate graph drawing and used the lined paper instead. As other candidates from the same schools did use graph paper it seemed to be a choice of individual candidates not to use the correct paper. For accurate graph drawing a mark could have been lost in this case.

Where clear working was shown, follow-through and method marks could be awarded when the answers were incorrect. A number lost an accuracy mark for the paper for not giving answers correct to 3 significant figures where appropriate.

# The areas of the programme and examination that appeared difficult for the candidates

Candidates experienced difficulties in:

- calculating both compound and conditional probabilities,
- drawing a Venn diagram with a subset,
- translating set notation into words,
- finding the equation of a line using integer coefficients,
- using a given scale to plot accurate graphs,
- understanding the limits of a regression line,
- identifying an angle in a three-dimensional diagram
- differentiating variables with negative indices.

The reasoning or showing a result questions also proved to be difficult.

# The areas of the programme and examination in which candidates appeared well prepared

The following topics were all well answered by a number of candidates:

- drawing tree diagrams,
- answering simple probability questions,
- finding the gradient of a line,
- finding the midpoint of a line,
- calculating the length of a line,
- writing down the mean and standard deviation from given data values,
- finding the correlation coefficient and the equation of the regression line,
- calculating a term and the sum of terms of an arithmetic sequence,
- using the sine and cosine rules.

It was encouraging to see the amount of working shown by candidates indicating that examination techniques had been taught.

# The strengths and weaknesses of the candidates in the treatment of individual questions

#### Question 1 Probability, Sets and Venn diagram

The majority of the candidates were able to draw the correct probability tree diagram and fill in the correct values. Most could also answer the simple probability question but many could not answer the compound probability correctly. The conditional probability question was not well answered with few candidates managing to recognize that indeed it was a conditional type.

Many candidates could draw the correct Venn diagram with the subset form and they could then go on and shade in the correct region. For those who drew an incorrect diagram follow-through marks were awarded for their shaded region, if it was correct. A significant number found it difficult to write in words the meaning of the shaded region.

#### Question 2 Coordinate Geometry

Most candidates could find the gradient of the line but some forgot the negative sign. If they showed working subsequently they could be awarded follow-through marks in the later parts of the question.

Most could find the midpoint of the line and could calculate the length of a line. Many could also find the equation of the required line but very few could give it in the required form. The question was asking them to give the equation with integer coefficients and from the candidates' answers it seemed as if they did not realize what was being asked. The final part of the question was well answered by a number of candidates with clear reasons being given but many could not give a clear mathematical reason why the lines were perpendicular.

#### Question 3 Statistics

This question was well answered by many candidates but a concern from the examination team was that some did not realize that they were expected to use their calculator to work out the mean and the standard deviation, the correlation coefficient and the equation of the regression line, and so lost time in calculating these manually. Many could not plot the points correctly on a graph although most could get the mark for the correct scale with named axes. There were many who plotted large points which covered one whole square on the graph paper. Only a few candidates wrote down the incorrect standard deviation and they could be awarded follow-through marks later. Commenting on the correlation coefficient was very well answered with most candidates giving both strong and positive for the two marks. The drawing of the regression line on the graph was not well done in a number of cases. It must pass through the mean point, which had already been plotted, and must pass through the correct y intercept which the candidates already knew from the equation of the regression line. Many lost one of the two marks for this part. Very few candidates could give the correct answer for the final part of this question as they did not realize that it was asking about extrapolation. Most calculated a value and said that their value was different.

#### Question 4 Arithmetic Sequences and Trigonometry

This question was well answered by many candidates except for the first part. Very few could equate the common differences to solve a linear equation. However, the rest of this part of the question was well answered with many achieving maximum marks. The second part of the question was well answered by a number of candidates. Only a few could not use Pythagoras theorem correctly to find the length of XO and they could be awarded follow through marks in later parts provided working was shown. Most could find the size of the required angle and then could go on to calculate the required length using the cosine rule; however, a number found difficulty in recognizing the required angle in the final part of the question.

#### Question 5 Calculus and Accurate Graph Drawing

Very few candidates could show the required expression in the first part of this question but they could then go on and use this for the rest of the question. Although negative indices are now on the syllabus there were many candidates who could not differentiate the expression correctly. Those who could differentiate the expression could then put the derivative equal to zero to find the value of x. Where working was shown follow-through marks were awarded for correctly putting their expression for the derivative equal to zero. The majority of candidates could find the values of a and b and plot the graph accurately. Many could draw a smooth curve through the points. However, some lost a mark in not seeing that the graph had a specific scale. Many found the last part difficult in finding the values of x for which y increases.

# **Recommendations and guidance for the teaching of future candidates**

It was pleasing to see many scripts with good, neat working shown. Candidates must be encouraged to write answers in ink however, (apart from drawing graphs in pencil) and begin each question on a separate sheet of paper. There were a number of scripts which were very difficult to mark as the candidates had written each part of a question on the same line with separate questions following on the next line. The answers should be spaced out more.

All of the syllabus should be taught and students need to be given experience writing tests in timed examination conditions.

Students should have as much practice as possible in answering questions written in different styles.

Students should have access to questions that ask for reasons or to show a result. They should clearly understand what is required in questions such as these.

Teachers need to remind students to give answers to the accuracy required in a question, or to 3 significant figures otherwise. Many candidates lost one mark for an accuracy penalty. All of the syllabus should be taught and special attention should be paid to the changes that have occurred in the syllabus.

Students should be given practice in drawing accurate graphs with non standard scales.

Students should be encouraged to look at answers and decide if they are reasonable.

The graphical calculator is now compulsory for this subject. Teachers must inform the students on how best to use the calculator in answering different types of questions. Specifically if a question part is only worth 1 or 2 marks then it might suggest that the calculator is to be used to find the answer. The command terms should be an indication of what is required in an answer. 'Write down' indicates that no working need be shown whereas 'calculate' indicates that some working should be given.