May 2016 subject reports

Mathematical Studies SL – Timezone 2

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2016 examination session the IB has produced time zone variants of Mathematical Studies SL papers.

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0–16	17–30	31–42	43–55	56–68	69–80	81–100

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–4	5–6	7–8	9–11	12–14	15–16	17–20

The range and suitability of the work submitted

There appeared to be a decline in the standard of work seen this session with a disturbing number of incomplete projects. Candidates opted almost unanimously for a statistical analysis. There is a bit of a paradox when candidates have for their title "Is there a correlation between ….". Surely anything other than correlation is irrelevant. Putting "Relationship" in the title instead would allow for more flexibility. It was pleasing to see that many candidates were aware that they needed two simple and one further process. However, when an error was made in one of the simple processes then they lost marks which was a shame given their overall standard. Including a third simple and relevant process can be a safeguard for achieving a higher level in this criterion. It would be nice for teachers to steer their candidates away from the obvious into a meatier investigation. Most of the samples from the schools had the full range of marks. If marks were below 5 then it was usually because the project was incomplete. Data collection was generally by questionnaire or internet sources (which were not always quoted). Unfortunately, there were still careless errors in calculations, notation and terminology and often variables were not defined.

Candidate performance against each criterion

A: The stronger candidates spent time on Criterion A to set up a framework for progression that allowed them to successfully address subsequent criteria successfully. Weaker ones did not establish this platform and struggled thereafter. Candidates generally were able to achieve level 2. Often candidates mentioned the mathematical processes that they would use but did not justify the reason for choosing each of the processes carried out. Occasionally processes not mentioned in the plan were carried out in the analysis or processes mentioned in the plan were not carried out. To be awarded level 3 there should be no surprises when reading the project.

B: In general, candidates understood this criterion well. Many candidates were able to achieve level 2 since the data collected was sufficient and organized ready for analysis. At times the data was limited or the quality was not good. Most candidates did not describe the sampling process. Phrases such as “I chose at random 50 participants” were often seen. Much more focus on sampling is needed. Only the very best projects included any details of the sampling technique selected. Some candidates needlessly threw away marks by failing to include their raw data.

C: Most candidates were able to perform some relevant mathematical analysis but there was not a wide range of techniques. Quite a few of the candidates used at least two simple processes along with a further process. At times the simple processes were not relevant to the task and this limited the award to level 2. Candidates often showed insufficient calculations in the simple processes and did not quote the formula they were using and calculator generated results appeared without working or interpretation and this made it difficult to assess understanding. The most common further processes were the χ^2 test and the correlation coefficient and equation of the regression line. Some candidates found the equation of the regression line before the correlation coefficient and often the equation of the regression line was not used. Some candidates found the regression line even although their value for r was weak. In some schools the candidates knew that they had to use Yates continuity correction when the degree of freedom was 1. In other schools they did not. Many candidates had expected values less than 5 and made no attempt to regroup their data. Some teachers ignored the fact that, if there are no simple processes in the project, then the first two further processes are counted as simple. Results were sometimes copied directly from the GDC with no explanation. This makes it difficult for the moderator to assess the level of understanding. Sometimes the processes were out of context with the aim and therefore not relevant. Other times the projects contained arithmetical errors which limits the possible score for this criterion.

D: Nearly all the candidates drew at least one conclusion from their results. However, some inconsistencies marred a few of the interpretations. Some candidates did not score highly in this criterion because the projects were too simple in conception to allow for substantive discussion. The stronger candidates had quite detailed discussion of their results. The project reads well if partial interpretations are written after each mathematical process. Candidates should be discouraged from making unsubstantiated conjectures about the reasons for their findings as these sweeping generalizations detract from the project.

E: This criterion is still the least well addressed. Some candidates made no attempt to fulfil this criterion. However, quite a few did comment meaningfully upon the processes used and the results found or they discussed the limitations of their results. Candidates think that their processes are valid if they have checked their calculations or they have performed their analysis on Excel. It was common for valid and accurate to be treated as synonyms.

F: Overall the projects were generally well structured and logically presented. A few of the projects did not contain comments throughout the task and this detracted from communication. Some candidates gave bibliographies and referenced sources. Commitment was lacking in some projects as some were too short and lacked mathematical analysis. Photographs of work done on paper should be discouraged as the projects will have better presentation if the work is typed and graphing software used.

G: Most candidates were able to earn one of the two marks for this criterion but few earned both. Terminology is sloppy and vague and variables are often not defined. Candidates should be taught how to use a simple equation editor. Many candidates are not using the correct symbol for χ or for multiplication. Some candidates still refer to “finding a correlation” rather than a relationship with reference to the χ^2 test.

Recommendations for the teaching of future candidates

- Read the subject reports.
- Encourage candidates to fully explain the reasons for using mathematical processes described in their plan.
- Ensure that the simple processes are meaningful and relevant to the task.
- Encourage candidates to show calculations that lead up to the result.
- Emphasize the importance of defining the variables.
- Emphasize the importance of clearly explaining any sampling process.
- Make sure that candidates include ALL raw data.
- Have candidates assess previous projects so that they understand the assessment criteria.
- Encourage candidates to use a different range of topics.
- Give candidates suggestions about how to increase the sophistication of their analysis.
- Give candidates the opportunity to correct errors in calculation and notation.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–12	13–25	26–36	37–48	49–61	62–73	74–90

General comments

The areas of the programme and examination which appeared difficult for the candidates

Some candidates were unable to classify numbers (natural, integer, rational and real) whereas others did this very well. Candidates did not read the question carefully for example failing to find the obtuse exterior angle of a triangle. Probability of combined events and conditional probability was not well understood and there were a number of candidates that gave an answer of greater than one for probability. In the χ^2 test candidates were unable to interpret the p-value found on their calculator and did not make a numerical comparison of this with the given significance level. The sketching of asymptotes often lacked accuracy. Few candidates were able to write a linear equation in standard form, possibly because they confused the meaning of integer with rational number. Some candidates were unable to solve an exponential equation; other candidates found the solution correctly using their GDC. Given the estimated mean of grouped data many candidates were unable to find a missing value in the frequency table. Most candidates were unable to use differential calculus to find when a cubic function had a specified gradient.

The areas of the programme and examination in which candidates appeared well prepared

All but the weakest candidates were able to rewrite a number in scientific notation. Some candidates were able to correctly classify numbers (natural, integer, rational and real) whereas others did this very poorly. Most candidates were able to use Pythagoras' theorem to find the height of a right angled triangle. Candidates did well in the trigonometry questions (using trigonometry ratios in right angle triangles as well as substituting into the law of sines and cosines). Candidates were able to fill in logic tables (though not always correctly) and write in words a compound proposition. Candidates successfully used the line of best fit to make predictions. Candidates were able to convert currencies. Linear functions were well understood and most were able to find parallel and perpendicular gradients and then find the equation of the line in gradient-intercept form. Most candidates could find the initial population given the exponential model. Some candidates found the solution of the exponential function correctly using their GDC but others were unable to solve the exponential equation. From a table of grouped data many candidates could identify the modal and median classes. Many candidates correctly differentiated the cubic equation.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Surface area of a sphere; scientific notation and percentage.

The weakest candidates were unable to square a number given in scientific notation or write the answer in scientific notation. Weaker candidates used the area of a circle formula rather than the surface area of a sphere. Premature rounding caused some candidates to obtain an incorrect final answer. Many candidates confused percentage of a quantity with percentage error or found the reciprocal of the correct answer. Overall this question was well attempted.

Question 2: Classification of numbers.

Stronger candidates were able to correctly identify if a number was rational, real or natural with the weaker candidates not recognizing that all rational numbers are real or perhaps these candidates lacked familiarity with the mathematical notation. Only the best candidates knew that $\frac{2}{3} \in$ ¤ , $\sqrt{2} \notin$ ¤ and that $-2^2 \notin$ ¥ but $-2^2 \in$ ¢ .

Question 3: Right angle trigonometry.

Candidates sketched the ladder leaning against the wall and recognized that Pythagoras' theorem was needed to find the distance between the top of the ladder and the base of the wall (but not always correctly). Although it was a right triangle a number of the candidates used the law of sines (instead of Pythagoras' theorem) and law of cosines (instead of a trigonometry ratio). Many candidates failed to find the obtuse angle made by the ladder with the ground even though the word obtuse was in bold type in the question.

Question 4: Logic.

All candidates recognized that to fill in a truth table the answer is either true or false. However, given that there are truth tables in the formula booklet it was surprising that some candidates made mistakes when negating a given column of the truth table. Most candidates recognized that in a tautology the column is always true with a small minority confusing tautology and contradiction. Candidates were able to write a compound proposition in words.

Question 5: Probability.

Some candidates confused the probability of both events occurring with the probability that one or the other occurs. Many candidates were unable to find the conditional probability. Candidates should not answer a probability question with an answer that exceeds one. Only the very best candidates did very well on this question; many found this to be one of the most challenging questions in the paper.

Question 6: Non-right angle trigonometry.

Instead of using the law of cosines weaker candidates substituted into Pythagoras' theorem and likewise used $A=\frac{1}{2}bh$ instead of $A=\frac{1}{2}ab\sin C$. Those that did select the correct formula almost always made correct substitutions but were not always able to calculate the correct answer.

Question 7: χ^2 test.

Candidates used their GDC to find the expected frequency with varying success whereas the p-value of the χ^2 test was usually correct; with some losing as many as four marks for giving answers to 1 significant figure with no working. As in the specimen paper the null hypotheses was not stated and so it was necessary to state what was being rejected. Candidates should write an explicit numerical comparison between p value and significance level to justify whether the null hypothesis is rejected or not. Amongst the candidates that made a comparison often the inequality sign was the wrong direction or the candidate made an inconsistent conclusion. There were many instances of poor mathematical terminology with correlation and independence used interchangeably likewise when candidates compared the significance level with their calculated χ^2 value.

Question 8: Rational function.

Few candidates could find the x-intercept of the rational function. Many candidates did appreciate that the curve does not cross the asymptote. Often the candidates wrote down the equation of the horizontal asymptote rather than the equation of the vertical asymptote. The most frequent incorrect sketch was that of $y=\frac{1}{2}x+1$ suggesting that the candidate did not understand that the curve $y=1+\frac{1}{2x}$ is not linear and had taken insufficient care in entering the function into the calculator. Some candidates that appreciated the shape of the curve did not earn marks on account of the poor quality of their sketches, which either crossed, or veered away from, the asymptotes.

Question 9: Linear regression.

The correct means were usually written down. Many candidates drew a line of best fit that did not go through their $(\overline{x}, \overline{y})$. Almost all candidates were able to use the line of best fit (either the one they had drawn or the regression line found using their GDC) to make a reasonable estimate. Feedback from teachers suggests that many are using line of best fit and line of regression as synonyms. This is not the case; both are explicitly mentioned in the guide and candidates are expected to understand both terms.

Question 10: Currency conversion and compound interest.

Currency conversion was done well by all but the weakest candidates. Most of the candidates that used the compound interest formula did a correct substitution but some did not equate this to the future value and found solving an equation to be challenging. Candidates that used the financial application on their GDC almost always wrote down a correct unrounded answer.

Question 11: Cylinder base area and curved surface area.

In responses to this question, units were sometimes missing or the wrong units were given. The question explicitly asked for the base and curved surface area but many gave both the top and bottom as well as the curved surface area, or omitted the ends.

Question 12: Linear function.

Many candidates demonstrated a good understanding of linear functions so successfully found the y-intercepts, gradient and equation in the form $y = mx + c$. However only the very best were able to rewrite this in the form $ax + by + d = 0$ where a, b and d are integers.

Question 13: Exponential model.

Most candidates were able to correctly substitute values into the given exponential model but only the stronger ones found a correct answer. It was expected that candidates would use their calculator to solve the exponential equation rather than use logarithms which is not in the syllabus. The concept of the population stabilizing (horizontal asymptote) was not widely understood.

Question 14: Grouped frequency table.

Candidates were able to identify the modal class and the class in which the median lies but few were able to find a missing value from the grouped frequency table given the estimated mean.

Question 15: Differential calculus.

Many candidates correctly differentiated the cubic equation. Most candidates were unable to use differential calculus to find the point where a cubic function had a specified gradient.

Recommendations and guidance for the teaching of future candidates

Show working

Although in paper 1 a correct answer without working is awarded full marks it is important to show working. If the candidate does not show working and makes a mistake in entering the values into the calculator or when writing a final answer, then all marks are lost.

Do not round prematurely

The final answer mark may not be awarded if a candidate uses a rounded intermediate value, for example 3.14 or $\frac{22}{7}$ as an approximation of π. Candidates should make use of the GDC's ability to carry forward a full answer to be used in subsequent steps.

Use the formula booklet

The formula booklet should be used throughout the two year course. Some candidates wrote incorrect versions of printed formulae.

Keep working visible

Some candidates erase pencil written working or cross out working without replacing it. Crossed out work is ignored. Answers written outside of the answer box, for example in the margins or amongst the questions, might not be seen by the examiner.

Understand mathematical terminology

If the question states "write down the equation…" the equality symbol and both sides of the equation must be seen; all too often candidates write an expression despite being asked for an equation.

Too many candidates made a conclusion for the χ^2 test by comparing the given 5% significance level to the calculated χ^2 value found on their GDC.

Be aware of common mistakes

In question 5 (probability), a candidate that gives a probability greater than one does not understand that $0 \le \text{probability} \le 1$.

In question 7 (χ^2 test), many candidates did not numerically compare the significance level with their p-value and draw a consistent conclusion.

In question 9 (linear regression), many candidates drew a line of best fit that did not go through their mean.

Practise past papers

Candidates need to be provided with practice in: a variety of probability questions; giving answers to the specified level of accuracy; writing a linear function in the form of $ax + by + d = 0$; drawing asymptotes and graphing functions with the GDC.

While an attempt is made to put the questions in order of difficulty it is suggested that candidates start by reading the paper and decide which are easier questions for them and work through these questions first.

Calculators

It is important candidates can use their calculators efficiently and therefore should be using their calculators regularly in class to ensure familiarity.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–28	29–41	42–51	52–61	62–71	72–90

General comments

This paper appeared to be accessible to most candidates. The variety of questions and level of difficulties of this paper provided opportunities to candidates to demonstrate their knowledge and understanding of the course. They were able to select and apply the different concepts that were being examined. Effective use of the GDC was noted. It was pleasing that the incorrect use of radians was almost never seen. In general, answers were accompanied by their appropriate units. It was pleasing to see that the majority of candidates were showing their workings. As a result, examiners awarded follow-through marks whenever applicable. However, many candidates seemed unsure about exactly how to interpret the command terms “sketch”, “draw” and “show that”.

Comments on the teacher feedback forms confirmed the appropriateness of the level of difficulty of this paper.

The areas of the programme and examination which appeared difficult for the candidates

Many candidates found it difficult to place numbers in the correct regions in the Venn diagram. They could not recognize the difference between the n^{th} term and sum of n terms in the context of a problem. Furthermore, understanding restrictions on domain and inequalities seemed to be difficult. It was surprising to see that many candidates were not able to use the correct window when sketching a graph. They did not always make good use of the graphic display calculator to help them draw graphs. Drawing a boxplot with an axis and correct label, knowing when to sketch versus draw appeared difficult. Candidates were not always successful at the “show that” question and at finding the maximum of a function using the derivative $V'=0$.

The areas of the programme and examination in which candidates appeared well prepared

The majority of candidates were successful at interpreting the values in the Venn diagram. They seemed to be quite comfortable with simple probability, finding the n^{th} term of both an arithmetic and a geometric sequence in the context of the problem. Furthermore, it was pleasing to see the number of candidates who were successful at sketching the normal curve, and finding the probability in the normal distribution problem. The majority of the candidates were able to use the sine rule to calculate the required sides and angles. They were also able to calculate the volumes of three dimensional solids with the correct units, to differentiate functions and to work with the cumulative frequency graph and quartiles.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Sets and probability

In part (a), a surprising number of candidates could not construct the Venn diagram correctly, based on the given information. This led to problems with the rest of the parts although they were usually awarded follow-through marks in part (b). Part (b) which required interpreting the information from their Venn diagram was generally well done. Some candidates gave the probability rather than number of people. Most candidates were successful at the simple probability but many struggled with the conditional probability.

Question 2: Arithmetic and geometric sequences and series

Parts (a), (b), (c) and (e) were well done. Quite a few forgot to convert their answer to km in part (c). The main problem with part (d) was that candidates chose to equate the n^{th} term formula to 1800 rather than the sum of the first n terms formula. Some of those who managed to write the correct equation were not always successful at solving it. Some candidates made use of the trial and error method to reach the correct answer. Part (e) was obvious to some, others put it into a formula with little understanding and a surprising number of candidates had place value issues (stating 10% of 17000 was 170). Many candidates used the compound interest formula in both parts (e) and (f). In part (f) many candidates did not realize that they needed to use the sum of a geometric series formula. They either used the sum of an arithmetic series or as previously mentioned, the compound interest formula.

Question 3: The normal distribution

Candidates showed comprehensive understanding of the normal distribution. The graphic display calculator was used efficiently by most of the candidates. There was much variability in the ability to sketch the curve in part (a). Instead of drawing the straight-forward sketch with the mean line and two vertical lines as required at 60 and 70, many linked it to standard deviations. It was very rare to see any method in part (c). Most candidates managed part (d)(i) but few went on to complete part (d)(ii).

Question 4: Trigonometry and volumes of 3D solids

This question was done well by most candidates. Trigonometry was a real strength with competent use of the sine rule. A small minority treated CB as parallel to AB and hence used alternate angles. The lack of a diagram in part (c) held some candidates back as they struggled to form the correct trigonometric ratio. Percentage error in part (d) was generally good. Most candidates scored the two marks as their answer to part (c) was followed through in part (d). Some candidates are still giving negative answers to percentage error problems. The common mistake in this part was the use of the new value in the denominator rather than the original value. Part (f) was less successful, in general, with a number of candidates not able to do the conversion.

Question 5: Differential calculus

In general, many candidates struggled in some parts. Most candidates who could state the dimensions of the box gave a reasonable justification of why x could not be 5. Very few candidates scored the two marks in part (d)(ii). Either their inequalities were not strict or their limits were incorrect or both. Some candidates stated the range of x as 1, 2, 3, 4. The algebra in part (c) caused problems for a number of candidates. It seemed that there was a lack of understanding of what the question required. Some substituted $x = 2$ in the volume formula. A few candidates wrote the product of the length, width, height, omitting the appropriate brackets. Part (d) was well answered by most candidates. However, its application in the following part was not as good in part (e). In part (e) some candidates left both solutions for x, not appreciating the fact that one was outside the range. Others lost both marks as they did not show that they had used their derivative to part (d) as required by the question. Very few candidates scored full marks for the sketch in part (g). Not following the given instructions about the domain and range let most candidates down in this question.

Question 6: Descriptive statistics

This question was, in general, well answered. Parts (a) to (e) were done satisfactorily. A lack of precision was noted in parts (b), (c) and (e). The drawing of the boxplot in part (f) caused problems to many candidates as far as labels and scale were concerned. The lack of scale on the graph paper made it difficult for examiners to mark their work. Some candidates used their answer booklet for drawing the box-and-whisker diagram, despite the instructions in the question to use graph paper. In part (g) the three quarters of 180 was successfully handled by most candidates but many divided it into 420 or 420–12 (rather than 420+12).

Recommendations and guidance for the teaching of future candidates

- It is important that candidates distinguish between a sketch and a drawing. Future candidates are encouraged to be accurate when sketching and drawing graphs.
- Accuracy with drawing the box-and-whisker diagram was an issue since candidates did not pay attention to the given instructions in the question. Future candidates are encouraged to use the scale given in the question. It is important that the drawing of graphs be done on graph paper. Labelling the graph must not be omitted.
- While constructing a box-and-whisker diagram, care should be taken that the whiskers do not cross the box.
- Candidates should understand how to place numbers in a Venn diagram; taking into consideration the intersection of the sets and the total number of elements in each set and the universal set is important.
- Candidates should follow the given instructions for each question and question part. For example, instructions about
    - the range and domain when drawing graphs
    - using a specific part to answer a question (the word "hence" is often used)
    - the degree of accuracy asked for
    - the units in which an answer must be given.
- Candidates should ensure that the question number and part are clearly written when that question part is being answered.
- Whenever more than one solution is given to the same part of a question, candidates should cross out so that their preferred answer is the one that is marked.
- Candidates are encouraged to reflect on their answers. They should make sure that their answer makes sense in the context of the problem.
- Candidates should know how to use and interpret the normal curve and the related x values and probabilities, e.g., finding x if $\mathrm{P}(X < x) = 0.9$
- It is important to show the stages of work, not just the final answer by using their calculator. It is to be noted that follow-through marks are only possible if working is shown.
- Candidates should avoid premature rounding off and hence avoid accumulating errors throughout a question, especially when the question part depends on the previous parts.
- Candidates should use the correct value for π, and not $\frac{22}{7}$ or 3.14.
- Candidates should follow the instructions to give answers exact or to 3 significant figures, unless otherwise stated in the question.
- It is important that candidates
    - recognize between an arithmetic series and a geometric series and thus apply the correct formula to solve problems
    - understand that the percentage error is an absolute value
    - know how to convert units
    - understand the requirements of a "show that" question