

May 2015 subject reports

Mathematical Studies SL Timezone 2

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2015 examination session the IB has produced time zone variants of Mathematical Studies SL papers.

Overall grade boundaries

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Standard level									
Grade:	1	2	3	4	5	6	7		
Mark range:	0–16	17–30	31–42	43–55	56–68	69–80	81–100		
Standard level internal assessment									
Component grade boundaries									

Grade:	1	2	3	4	5	6	7
Mark range:	0–4	5–6	7–8	9–11	12–14	15–16	17–20

The range and suitability of the work submitted

Compared to last year there was a much better grasp of the new criteria, especially the relevance of mathematical processes. Candidates seem to be reflecting more on why they are using certain processes in their project. It was pleasing to see that many candidates were aware that they needed two simple and one further process. Projects were much more focused and plans were well presented on the whole. If marks were below 5 then it was usually because the project was incomplete. Most of the candidates, as usual, opted for statistical analysis. Data collection was generally by questionnaire or internet sources (which were not always quoted). Unfortunately there were still careless errors in calculations, notation and terminology.



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Candidate performance against each criterion

A: Candidates generally were able to achieve level 2. Often candidates mentioned the mathematical processes that they would use but did not justify the reason for choosing each of the processes carried out. Occasionally processes not mentioned in the plan were carried out in the analysis or processes that were mentioned in the plan were not carried out. To be awarded level 3 there should be no surprises when reading the project.

B: Many candidates were able to achieve level 2 since the data collected was sufficient and organized ready for analysis. At times the data was limited or the quality was not good. Most candidates did not describe the sampling process. Phrases such as "I chose at random 50 participants" were often seen. Much more focus on sampling is needed. Only the very best projects included any details of the sampling technique selected. Some candidates needlessly threw away marks by the failure to include their raw data.

C: Quite a few of the candidates used at least two simple processes along with a further process. The most common further processes were the chi-squared test and the correlation coefficient. In some schools the candidates were aware that they needed to apply Yates Continuity correction when the degree of freedom was 1. In other schools they did not. Many candidates had expected values less than 5 and made no attempt to regroup their data. Some candidates found the regression line even although their value for r was weak. Some teachers ignored the fact that, if there are no simple processes in the project, the first two further processes are counted as simple. Results were sometimes copied directly from the GDC with no explanation. This makes it difficult for the moderator to assess the level of understanding. Sometimes the processes were out of context with the aim and therefore not relevant. Other times, the projects contained arithmetical errors which limited the possible score for this criterion.

D: Candidates were, on the whole, able to draw one conclusion from their results. The stronger candidates had quite a detailed discussion of their results. The project reads well if partial interpretations are written after each mathematical process. Some candidates still gave irrelevant or unsupported conclusions or wrote down their own personal beliefs.

E: Some candidates made no attempt to fulfil this criterion. However, quite a few did comment meaningfully upon the processes used and the results found or they discussed the limitations of their results.

F: Overall the projects had structure and they developed logically. Some candidates gave bibliographies and referenced sources. Commitment was lacking in some projects as some were too short and lacked mathematical analysis. Photographs of work done on paper should be discouraged as the projects will have better presentation if the work is typed and graphing software used.

G: Many candidates only scored one mark for this criterion. Many candidates are not using the correct symbol for χ or for multiplication. At times variables were not explicitly described. Some candidates still refer to "finding a correlation" rather than a relationship with reference to the χ^2 test.



Recommendations for the teaching of future candidates

- Read the subject reports.
- Encourage candidates to fully explain the reasons for using the mathematical processes that are described in their plan.
- Variables should be defined.
- Sampling should be explained more fully.
- Include ALL raw data.
- Have candidates assess previous projects so that they understand the assessment criteria.
- Encourage candidates to use a different range of topics.
- Encourage candidates to show calculations by hand.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–12	13–25	26–36	37–48	49–61	62–73	74–90

The areas of the programme and examination which appeared difficult for the candidates

- identifying an angle of depression,
- interpreting an estimate from a box-and-whisker diagram,
- complex compound propositions in logic,
- understanding the difference between *p*-values and χ^2 values,
- geometrical properties of isosceles triangles,
- compound interest,
- validity of logic statements,
- trigonometry of 3D shapes,
- Normal distribution and interpreting data given by the GDC,
- substituting into formulae containing functional notation,
- identifying a maximum and a range from a given mathematical model.



The areas of the programme and examination in which candidates appeared well prepared

- using trigonometric functions to find the sides of right-angled triangles,
- percentage error,
- equations of straight lines,
- drawing box-and-whisker diagrams,
- truth tables,
- identifying set structures,
- χ^2 test (although please note the comment in the previous section),
- identifying the nth term of a geometric series,
- simple logic.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Decimal places, significant figures and standard form

Despite a significant number of candidates scoring well on this first question, a minority of candidates failed to understand the demands of the question. In part (a), candidates were expected to give their answer to two decimal places. If their answer was the required answer of 224.96 there was no issue and these candidates were awarded 3 marks. Candidates who did not have the required answer and did not show a previously unrounded answer gained, at most, one mark in this part of the question. Despite some truncated 3 figure answers scoring no marks in part (b) this second part was done well by many candidates. A minority of candidates did not read the demand for part (c) correctly and wrote their answer to part (a) in standard form for their answer to this part of the question. Such candidates lost at least one mark here. However, it was encouraging to see that there was lots of good work for this part of the question showing that candidates are well prepared for questions on standard form.

Question 2: Trigonometry and percentage error

Many candidates did not seem to be able to identify the angle of depression on the diagram with angle $BTC = 30^{\circ}$ often erroneously seen. In part (b), lots of good trigonometry was seen and, with the aid of the follow through marking, many achieved full marks on this part of the question. It was pleasing to see that many centres had prepared their candidates well for percentage error problems. Indeed, very few candidates made the mistake of dividing by 150 and very few gave a negative answer showing that they understood the significance of the modulus sign in the formula.



Question 3: Equation of a straight line and gradients

Some very good responses here showing that candidates were well prepared for questions on calculating gradients and determining the equation of a straight line. A word of caution however, candidates should show their working at all times. Where an answer is correct, with or without working, full marks are awarded; however in part (c), where a follow through answer, however trivial, was given with no working shown, no marks could be awarded.

Question 4: Descriptive statistics

Simply writing down an answer of 5 rather than the correct answer of 4 (because 5 is the median of the set of data 2, 3, 48) earned no marks at all here and caused further problems to candidates in part (b) as their 5 coincided with Q₃. Despite this lack of understanding of the term *median*, many candidates did score full marks on parts (a) and (b). Part (c) proved to be more problematic as a significant number of candidates simply interpreted the phrase *at least* 6 *books* as being the proportion of candidates who had read up to and including 6 books. The decision to interpret this as 75% was seemed to be influenced somewhat by the fact that Q₃ is the 75th percentile. Fewer candidates than expected gave the correct answer of 10 here, with many giving the incorrect answer of 30.

Question 5: Logic, truth tables

Although this question was done well by the majority of candidates, there were still a significant number of candidates who, despite standard truth tables being given in the formula booklet, made errors in table entries in part (a). Part (b) proved to be quite a discriminator as many candidates focused their answers by looking at the columns of the components in the equivalence rather than the full statement. As a consequence, many incorrect answers of the form "the columns have a mixture of Ts and Fs therefore it is neither" were seen. Successful candidates for invariably drew uр а column the compound proposition $\neg((r \land p) \lor \neg q) \Leftrightarrow \neg(r \land p) \land q$. If this was correct from their table entries then the reasoning mark was awarded. The final mark was awarded as a follow though from their reasoning mark. (It is important here to state that without reasoning, no marks can be given for this type of question.)

Question 6: Sets

The most common error seen here was getting the first two table entries the wrong way round. Otherwise, many candidates scored at least 3 marks on this question.



Question 7: χ^2 test for independence.

In part (a), the majority of candidates seemed to be well drilled in using the word *independence* in a Null Hypothesis. A minority of candidates, however, seemed to be confused as to what was independent and some scripts identified, incorrectly, that gender, rather than age, was independent from the preferred type of dance. Many correct values were seen in parts (b) and (c) although some candidates gave the expected value of 14 in part (b) and therefore lost this mark and, in part (c), truncated their *p*-value to 0.0875 losing one of the two marks as a consequence. Candidates should be advised to give **all** figures from their calculator display in their working. Part (d) was specifically testing the candidate's understanding of the relevance of the significance value to their *p*-value; had the intention of the question been for the candidates to compare the critical value against the χ^2 statistic, the critical value would have been given. Many candidates drew correct conclusions using their *p*-value and the 5% level of significance in this part of the question. However some candidates seemed to be confused between the χ^2 statistic and the *p*-value and tried to compare these, losing both marks as a consequence.

Question 8: Trigonometry and geometry of none right-angled triangles

Many correct answers of 10 m were seen in part (a) but this was then spoiled by the majority of candidates as many did not seem to know the basic properties of isosceles triangles. As a consequence, incorrect working and incorrect assumptions were seen in part (b) culminating in a variety of answers which were not 110° . Using their angle found in part (b), many candidates were, however, then able to use the cosine or sine rule to arrive at a correct follow through answer for part (c). Completely correct solutions were rare here.

Question 9: Arithmetic and geometric series

Too many candidates identified incorrect sequences in part (a) and, despite the initial line in the question stating *only one of the following...*, some candidates gave multiple answers. Although a significant minority of candidates wrote the answer 2 for part (b), the vast majority correctly wrote down the common ratio of 0.5. Although the sum to *n* terms of a geometric sequence was seen on a few scripts, many candidates wrote down the correct formula for the 10^{th} term of the series using their common ratio for two marks in part (c). The final mark proved to be a little more elusive as the **exact** fraction was required. Many simply left the decimal answer of -0.01171875 or gave the three significant figure version of this value. Both lost the final mark as the correct fraction was required. Candidates who had an answer of 2 for part (b), also lost the final mark here by giving the answer of -3072.



Question 10: Finance

Despite the acceptance of using the Financial App on the graphic display calculator, (and awarding marks accordingly), very few candidates went down this route with the overwhelming majority of candidates writing down a compound interest formula. In part (a), many candidates substituted correctly into the compound interest formula to arrive at the required answer of 1160.75. Many of the incorrect substitutions were either as a result of misinterpreting *interest rate of 5%* as 0.5 rather than 5 in the formula or misinterpreting *compounded quarterly* as implying that there must be a 3 periods in the year (rather than 4) in the formula. Some guess work and trial and error was seen in part (b) as many seemed unable to set up the required equation. Indeed, this question proved to be quite a discriminator as even those candidates who set up the correct equation often were unable to solve their equation to arrive at the required answer of 5.28 years.

Question 11: Symbolic logic

Many candidates scored well on the first three parts of this question. Unfortunately, in part (d), candidates had little understanding of the validity concept in context. Many tried to use truth tables and a significant number mixed up the concept of factors with the concept of multiples. A counterexample (e.g. 18 is a multiple of 6 but not a multiple of 12) showing that the statement is invalid was seen on only a minority of scripts.

Question 12: Trigonometry and geometry of 3D solids

The majority of candidates were able to write down the required answer of 4 cm for part (a). Part (b) proved to be a little more problematic, especially for those candidates who attempted to work out the height, VO, of the pyramid. A correct height, rounded to 3 significant figures and used correctly in a trigonometrical ratio or in the cosine formula, invariably led to an answer of 66.5° losing the accuracy mark. An incorrect use of Pythagoras' Theorem leading to a height of 10.8 cm lost both marks here. Despite errors in part (b), many knew how to apply the area of a triangle formula in part (c) but, in a number of cases, only found half the required area. It was pleasing to see that the majority of candidates remembered to put units (cm²) after their numerical answer.



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Question 13: Normal distribution

A lot of correct answers were seen in part (a) as the correct shading to the left of a vertical line labelled 175 cm was seen on many scripts. Part (b), however, proved to be too much of a challenge. The wording of the two parts of the question proved confusing with many candidates thinking they required a probability for part (i) rather than a height and 0.159 proved to be a popular, but incorrect, answer. Many of these candidates had nowhere to go in part (ii) and the answer line was often left blank. Of those candidates who did recognize that, for part (ii) they required 180 + 5 = 185 cm, a large number found, without working, the required answer. A cautionary note here however is that simply showing the GDC commands earned no method here and an incorrect answer earned no marks.

Question 14: Mathematical modelling, exponential model

Many candidates seemed to misunderstand the notation here, interpreting N(x) as $N \times x$. This error was often compounded by the candidate's inability to handle the negative index. As a consequence, this question was poorly answered. In part (a), *t* was often substituted with the value 1 so $ab^1 = 800$ often led to a final answer of $a = \frac{800}{b}$ earning no marks at all. Many candidates fared no better in part (b) with 360 (90×4) often being equated to the given function. Rearranging their equation and removing the negative index of *b* also proved problematic and few, if any marks were earned in part (b). Recognizing the asymptotic value of 40 was beyond most candidates in part (c) and as a consequence, few, if any, marks were achieved by the majority of candidates in this question.

Question 15: Quadratic function

Most candidates achieved at least the first two marks for this question as they equated their answer to part (a) to the quadratic function. Solving their equation however proved to be problematic and the required answer of 180 m was not seen as frequently as one might expect. The demand for part (c)(i) did not seem to be fully understood by many candidates as 180 (which is the maximum value of *x*) was given as the answer rather than 100 which could have been arrived at by the symmetry property of the quadratic. Despite this error, many were able to recover in part (ii). Finding the range of *A* proved to be quite problematic with very few candidates realizing that the important values were their answers to part (a) and to part (c)(ii) and many incorrect or missing answers were seen.



Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- Practise the correct use of functional notation. Interpreting f(x) as $f \times x$ should be shown as being incorrect.
- Unless demanded in the question, show all working and give answers to at least three significant figures. (Remember, follow through marks are generally not awarded if working is not seen.)
- Critically examine their answers to see whether or not they are sensible in the context of the problem set.
- Not cross out their work unless it is to be replaced crossed out working earns no marks at all.
- Label their work in the working box, to improve readability.
- Draw diagram(s) in questions on normal distributions shading appropriate areas where necessary.
- Practise more questions where a mathematical justification is required.
- Practise the use of the GDC for questions involving statistics, normal distribution and finance.
- Ensure that candidates are fully conversant with the formulae which appear in the information booklet and where exactly these formulae are to be found in the booklet, prior to the examination.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0–14	15–28	29–40	41–50	51–61	62–71	72–90

General comments

The paper seemed to be accessible. The majority of the candidates demonstrated good knowledge of the course material and the ability to apply that knowledge to answer the exam questions. Candidates were able to select the appropriate techniques to solve problems.



The areas of the programme and examination which appeared difficult for the candidates

- Substituting negative values in an expression and calculating the result
- Use of regression line
- Normal distribution
- Venn diagrams working backwards from totals
- Compound and conditional probability Identifying the "or" as indicating the union
- Identifying S_1 as u_1
- Solving a quadratic equation
- Use of the GDC to find mean and standard deviation
- Solving for *n* in the sum of an arithmetic series
- Substituting and squaring negative numbers
- Differentiation of negative powers
- Labelling graphs (x & y) and sketching smooth curves
- Equation of a tangent and graphical concept of tangent at a point

The areas of the programme and examination in which candidates appeared well prepared

- Simple probability
- Application of the Pythagoras' Theorem
- Areas and volumes including correct units for 2D and 3D
- Currency conversions
- The *n*th term of an arithmetic sequence
- Differentiation of positive powers
- Correlation coefficient
- Equation of regression line

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Venn diagram and probability

Part (a) was well answered by many candidates. Most candidates scored at least 2 marks for 15 and 8 in the correct regions. The common mistake was to write 33, 22 and 27 instead of 18, 7 and 12. Some candidates did not copy the Venn diagram and as a result lost the 3 marks. Very few did not include the rectangle in their diagram.

Part (b) was well answered by many candidates. Some candidates did not give their answers in the requested form, that is, in terms of x. They wrote the values of x and as such lost at most



1 mark if the correct values (10 and 20) were given in the correct regions. The common mistake was to use x and 0.5x.

The correct answer for part (c) was obtained by many candidates. Some candidates omitted 8 in their sum of the elements of their Venn diagram while solving for x.

Many correct answers were seen to part (d).

Parts (e)(i) and (ii) were accessible to candidates; few scored both marks in part (iii).

The most common error in part (f) was the use of $\frac{8}{100} \times \frac{8}{100}$ or the use of addition instead of multiplication. However, many candidates could answer this part.

Question 2: Volume and surface are of 3D solids

In part (a) many candidates applied Pythagoras' Theorem successfully. Some stopped at $\sqrt{7}$ or the full calculator value.

It was good to see that candidates were writing the correct units. Many candidates scored full marks in part (a)(ii).

Part (b) presented no major problems for candidates, with many scoring well. Some used 127 instead of 125.

Many candidates scored at least 2 marks in part (c). The common mistake to this part was the addition of the area of the base of the cylinder to both curved surface area of cylinder and of the cone.

Part (d) was very well attempted by most candidates. The follow through marking allowed candidates to score full marks in spite of an incorrect value in the previous part. However, it was noted that many candidates did not round off their final answer to the correct number of decimal places.

Part (e) was well attempted by most candidates.

Question 3: Arithmetic sequence

Part (a) was well answered.

For many candidates, part (b) proved to be difficult. They found it hard "to show" as they could not link the sum of the terms to the actual terms in the sequence. Many candidates made use 9 - which was to be shown – to find a common difference of 2 and then substituted those values into the arithmetic sequence formula.

Part (c) was well attempted.



International Baccalaureate Baccalauréat International Bachillerato Internacional In part (d) it was usual to see the correct formula substituted. Some candidates added their difference to the first term instead of multiplying it with 9.

Many candidates compared their nth term to 1001 instead of 1000. Many candidates used trial and error to find the value of n.

Few candidates were successful at part (f). The first method mark was easily obtained by the majority of candidates. The main difficulty seemed to have been the simplification and solving of the quadratic equation.

Question 4: Normal distribution

Not many candidates could score full marks in part (a). A commonly seen incorrect answer was 7 and 1.41.

Part (b) was well attempted. However, very few candidates showed the correct region on a normal distribution diagram.

Very few candidates could find the correct answer to part (c). This part appeared to be a difficult problem to solve.

In part (d), many candidates benefited from the follow through marking.

Part (e) was well attempted by the majority of candidates.

Question 5: Differentiation, equation of tangent

In part (a) substitution into the formula was well done by the majority showing good knowledge of function notation although sometimes the answers arrived at were incorrect, indicating some weakness in algebra.

In part (b) many correct answers were seen for the derivative. Some candidates did not simplify their answers.

In part (c) correct substitution was seen in the candidates' derivative. However, many candidates could not deal with the negative index.

Not many correct answers to part (d) were seen. There were several successful attempts at substituting their part (c) and part (a) into y = mx + c. Candidates benefited from the follow through marking.

It is important to highlight that, in part (e) many candidates drew the function instead of sketching it. Candidates should be aware of the differences between the command terms. Many candidates used graph paper for this part. Proper labels and scales were not often seen.

Very few candidates were successful at drawing a tangent at a point in part (f).



International Baccalaureate[®] Baccalauréat International Bachillerato Internacional In part (g) not many candidates could find the point of intersection of the function with the tangent. This showed a lack of efficient use of the GDC.

Question 6: Correlation and regression, modelling

The responses to part (a) were generally good. The main problem with this part was the incorrect rounding off of the value. Many candidates either stated "strong" or "positive" instead of both.

The correct equation of the line of regression was regularly seen in part (b).

Correct substitution of 13 into their regression line was seen in most candidates' part (c). Many candidates used the given table to estimate *y*. Some candidates did not give their answer to the required accuracy.

Part (d) was well answered in general. It was good to see proper workings to support the answer.

Part (e) was seen as difficult by many candidates. Not many of them could write the correct expressions in part (e). However, many correct answers were seen for part (e)(iii).

Recommendations and guidance for the teaching of future candidates

- Teachers should reinforce conditional probability, the concepts of sets and Venn diagram.
- Teachers should reinforce linear correlation and the normal distribution.
- Candidates should be encouraged to carefully read instructions and give answers to the correct accuracy as required.
- Candidates must practise sketching unfamiliar graphs with the help of their GDC. They must be encouraged to use the given domain and to analyse their graph for key features. This will lead to improved precision while sketching graphs.
- Encourage candidates to show all stages of work especially with "Show that" questions.
- Make it clear to candidates that omitting their workings may result in less marks being awarded. Candidates need to be reminded to make the most of the "show that" types, using these values to proceed with the other parts of the questions.
- Candidates should be better prepared in answering questions in specific contexts. They must be prepared to apply the concepts in different situational contexts.
- Candidates should reflect on the accuracy of their answers in the context of the problem.
- Prepare candidates to use problem solving techniques in unfamiliar situations.
- To avoid premature rounding off, which leads to a lack of accuracy in the final answer.

