MATHEMATICAL STUDIES TZ2

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 29	30 - 41	42 - 54	55 - 67	68 - 79	80 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2014 examination session the IB has produced time zone variants of Mathematical Studies papers.

Standard level Project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

There was a wide range of marks as usual. Most of the topics were statistical and were suitable for a Mathematical Studies SL project but there are always a few that should have been actively discouraged by the teachers to start with. Some candidates had obviously worked hard on their project and enjoyed the process and this was obvious from the care that was taken to satisfy all the assessment criteria and, as a result, these candidates scored highly on all the criteria. However, there were others that showed little, if any, commitment and produced a trivial or incomplete piece of work. Some schools did not realize that the projects had to include two simple processes first before a further process was attempted and these scored poorly on Criterion C. Many candidates lost a mark due to improper notation and/or terminology or failing to define variables and teachers should take more care to point this out to their students. Some teachers were still using the old criteria and the 5/PJCS for the previous syllabus. Such work was moderated against the correct May 2014 criteria. Teachers should refer to the current guide and the up-to-date handbook of procedures (where the 5/PJCS is saved). It is important that teachers write detailed comments on the front of the cover sheet explaining why the marks were awarded. They are also encouraged to make comments throughout the project in pencil in the margins and check the accuracy of the mathematics.

Candidate performance against each criterion

Criterion A:

Many candidates were awarded a level 2 out of a possible 3. This was mainly due to the fact that they did not give any reasons for the processes they were going to use.

Some candidates only scored 1 mark because their plan was not clear or their project had no title.

To award level 3 there should be no surprises when reading the project. For the plan to be considered detailed, the student should describe precisely all the mathematical processes to be used and the reasons for choosing each of these processes.

If any processes are used that are not discussed in the introduction then at most level 2 can be awarded.

If any process is explained in the introduction but not performed, then at most level 2 can be awarded.

Candidates with a clear statement of task and detailed plan discussing the processes to be used and the rationale behind their choices usually produced excellent projects.

Criterion B:

Most candidates collected data that was appropriate for their project but it was not always sufficient in quantity to perform the processes set out in their plan.

Few candidates described their sampling process clearly and so were not awarded full marks for this criterion.

The collection process must be thoroughly described and must be representative of the population. Saying that the data was randomly collected is insufficient. The sampling process must be explained. If sampling is not done then this must be justified.

If no real organization of the data is required then at most level 2 can be awarded for this criterion.

Raw data must be seen to consider level 2 for this criterion.

Calculations must be able to be checked.

Data that is too simple also limits the marks for other criteria such as the mathematical processes, interpretation and communication.

Criterion C:

Most of the changes in the new assessment criteria are in this criterion. Not all teachers and candidates paid attention to the changes and, as a result, did not score well.

The candidates must complete at least two simple processes that are correct and relevant to be awarded level 3 for this criterion. It is required that only all *simple* processes are relevant at this level. Irrelevant further processes do not preclude the candidate being awarded level 3.

Simple processes are considered relevant if they pertain to the statement of task and if these processes are used later in the development of further processes, as stated in their plan.

If there are no simple processes in the project, then two of the further processes will be considered to be simple processes and **not** further processes.

Repeated processes count as one process (e.g. producing two bar charts).

If the project includes only two processes and one is incorrect, then level 1 is the maximum which can be awarded.

If there is only one process used, simple or further, then the candidate is awarded level zero.

If the simple and further processes are not presented in order, the student will not be penalized in this criterion. However this may be penalized in criterion F.

To be awarded level 5 all further processes (and there only needs to be one) must be without error, and must be relevant.

Any process that is beyond the course needs to be fully explained to be considered a further process, for example the unsupported use of the *t*-test, whether performed wholly on the GDC or by substitution into the formula is deemed a simple process.

Although the processes are not limited to the chi-squared test and calculating the regression equation, the frequency with which they appear makes it worthwhile producing further guidance on how they should be marked.

Chi-squared test

A χ^2 test performed by hand is considered to be one further process.

For a completed χ^2 test candidates are expected to write down their hypotheses, degrees of freedom, show how to calculate at least one expected value and complete the table of expected values, work out the chi-squared test statistic using the formula and write down the conclusion (using either the critical value or the significance level).

If the observed values are not frequencies, then at most level 3 can be awarded for criterion C.

If any expected values are less than 5, then at most level 4 can be awarded for criterion C, and only if all the working is shown in full. If the working is not shown, then at most level 3 can be awarded.

If the degree of freedom is 1, then Yates continuity correction must be applied (and only when the degree of freedom is 1). If the correction factor is not applied and the test has been satisfactorily performed by hand then at most level 4 can be awarded.

Candidates should note that a χ^2 test does not prove anything. It supplies evidence or support only.

Correlation / regression

If the candidate draws a scatter diagram and it is clear from the diagram that there is no correlation then it is relevant to calculate the correlation coefficient, r , to verify that fact. However, it is not relevant to calculate the regression line.

If from the scatter diagram it seems that there is some correlation then it is relevant to calculate the correlation coefficient, r , and, if the correlation is strong enough, then it is relevant to find the regression line, provided it is used or its purpose explained.

If a scatter graph is not drawn, then the relevancy of a regression line will depend on the value of r .

If the value of r is written down from the GDC (or Excel) then this is a simple process.

If the summary statistics have been calculated from the GDC and then substituted into a formula to determine r this is also a simple process.

Calculation of the mean or standard deviation as part of calculating r is not considered a separate process. The exception to this is if the mean or standard deviation has been calculated independently as part of the stated plan.

Normal distribution

Sketching a normal distribution curve and calculating probabilities or percentages is a simple process.

Using z-scores is also a simple process.

If a χ^2 goodness of fit test is performed by hand, then this is a further process.

Criterion D:

The project flows better if the candidate writes partial interpretations/conclusions after each mathematical process.

Most candidates managed to give at least one interpretation that was consistent with their analysis. However, the wording in this criterion has now changed and, if there are any inconsistent conclusions/interpretations, then there must be at least two consistent conclusions/interpretations for the candidate to be awarded level 2.

Any irrelevant or unsupported conclusions (or personal beliefs) preclude the award of level 3.

Criterion E:

Many candidates now show more understanding of validity and are able to comment meaningfully on the mathematical processes used or recognize limitations and provide a discussion.

Recognizing and commenting on the need to use the Yates' continuity correction factor or combining groups in the χ^2 test is sufficient for this criterion.

Criterion F:

Overall the structure of the projects was good. However, this criterion covers more than the layout, it also deals with commitment. The project must demonstrate the required time commitment otherwise the maximum that can be awarded is level 1.

Some candidates included unsupported generalizations and this does not lead to a coherent project. Also, a large number of repetitive procedures preclude the award of level 3.

Graphs, tables or processes presented out of order also preclude the award of level 3.

If many pages of raw data or calculations via spreadsheet are presented, it is preferred that these be shown in an appendix; however this is not penalized.

If processes have been mentioned in the introduction and have not been performed or vice versa then the candidate is not penalized twice for the same error.

Criterion G:

Surprisingly few candidates scored full marks on this criterion. The most common level awarded was 1 due to incorrect notation and/or terminology or failure to define variables.

Candidates that use Excel or calculator screen dumps need to be aware that this notation is not acceptable. If there are examples of such notation this must be explained and corrected in the body of the text.

Candidates should avoid using their cameras to take pictures of calculator screens.

Isolated typographical errors are condoned, however if the candidate uses $x \wedge 2$ instead of x^2, for example, this is poor notation and the maximum that can be awarded is level 1.

Examples of notation:

Correct notation	**Incorrect notation**
x^2	$x \wedge 2$ **or** $x2$
$x \times 2$ **or** $2x$	$x * 2$
1.2×10^{-3}	1.2 E-03
χ^2	X^2 **or** x^2
r^2:Coefficient of determination	r^2:Correlation coefficient
$\sqrt{\frac{2402}{16}}$ **or** $\sqrt{}(2402/16)$	$\sqrt{}$ 2402/16 **or** sqrt.

Recommendations and guidance for the teaching of future candidates

- Read the Subject Report. This is extra important with the new set of criteria.
- Set internal deadlines for the project.
- Have students assess previous projects to gain an understanding of the assessment criteria.
- Encourage students to show calculations by hand even if they are making use of technology such as Excel.
- Help the students to understand how to address validity.
- Encourage the students to use at least two simple processes in their analysis.
- Make sure that the students define any variables in their project.
- Show the students how to use equation editor and where to find the symbol for χ.
- Show the students how to use Yates' continuity correction.
- Make sure that students attach all raw data.
- Explain sampling to the students.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 24	25 - 35	36 - 47	48 - 60	61 - 72	73 - 90

The areas of the programme and examination which appeared difficult for the candidates

- Candidates should acknowledge and check for the reasonableness of their answer. For example, in calculating the distance in kilometres that the Earth travels around the Sun in one orbit is not likely to be 942 km.
- Many candidates had problems with determining the correct set for a Venn diagram.
- Many candidates were unable to solve a quadratic equation and did not realize that a solution which gave a negative length was impossible.
- Candidates had problems in judging what working to present for reasoning questions and in presenting concise well-thought-out interpretation. For example, the candidates could not justify which exchange rate was optimal using a numerical comparison; frequently this question was left unanswered and those that did answer simply stated which exchange rate was the best without a justification. Another example was in a logical reasoning, where they were not able to articulate their answer in a coherent manner and a truth table would have been the best way to provide the justification of logical equivalence.
- Although the candidates were able to find the p-value or the χ^2 calculated value, there was confusion in how to interpret this.
- Candidates must read the instructions such as giving the answer to the nearest whole number.
- Many made the trigonometry much harder than was intended by missing the use of right-angle trig ratio formulae, and used instead the sine and cosine rules. Weaker students had difficulty finding the angle in a right-angle triangle using SOHCAHTOA rules.
- Although most candidates could set up an exponential equation, very few could solve for the exponent itself when it is unknown.
- Candidates confused local extrema with absolute extrema. For example, they labelled the minimum or maximum as presented on the graph given rather than the local minimum.
- Only the very best could identify the interval for which the function was decreasing and find the equation of a tangent when given both the equation and the graph.
- The candidates struggled with finding the derivative of 4/x.
- It was evident that some candidates had not studied the normal distribution.
- Often candidates did not show their workings and lost all marks in a question by writing down only an incorrect final answer.

The areas of the programme and examination in which candidates appeared well prepared

The majority of candidates appeared to finish the paper and attempt all the questions with correct units included as appropriate. The working was clearly shown on most papers so these candidates benefited from the follow through and method marks even though their final answer was incorrect. Candidates could substitute into formulae given in the formula booklet, write in words $q \Rightarrow p$, find the volume of a sphere, read values from a box and whisker plot, use an exponential equation to find values and use their calculator to find an answer such as the Pearson's product moment correlation co-efficient. Good candidates had little difficulty with the converting currencies, using right trigonometry to find a missing length and find the angle of elevation. Candidates did better in the questions not set in a context and which required no comments, explanation or justification.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: write a number in the form of *a* x 10^k; circumference of a circle; multiplication

The question asked the candidates to write 150 million in the form of *a* x 10^k; many ignored the word "million" and obtained an unrealistic distance for the orbit of the Earth around the Sun. Weaker candidates confused circumference with area. Occasionally candidates wrote "1.6E10", copied from their calculator, but this is not acceptable mathematical notation

Question 2: write in words $q \Rightarrow p$ and the contrapositive; logical equivalence

Some candidates missed the words "if... then". Those that used truth tables to determine logical equivalence answered this question well. However many wrote meaningless wordy explanations.

Question 3: using a Venn diagram to classify types of numbers

Surprisingly few candidates could place all the numbers in the correct position on the Venn diagram. For example, as 1/3 is a recurring decimal, many placed this as an irrational number.

Question 4: volume of a sphere and box; arithmetic

This was very well done in general, including the units which were either correct or omitted (few candidates used incorrect units). Most candidates found the volume of one chocolate although some used an approximate answer for π. The volume of the box proved more difficult and a common mistake was 10 cm x 8 cm x 1 cm rather than 10 cm x 8 cm x 2 cm. This error led to a negative volume for the last part, although these candidates chose to ignore the negative sign.

Question 5: box and whisker diagram

Many were unable to find the number of days between 43 mm (median) and 48 mm (upper quartile) from a box and whisker plot and did not realize they needed to find 25% of 80 days to obtain the correct answer.

Question 6: area of a rectangle and solving a quadratic

The most common mistake was to give area of the rectangle as $2x\,x-4$ rather than $2x\,(x-4)$. Only the very best were able to solve the quadratic equation and discard the answer that was a negative length.

Question 7: finding gradient and *x*-intercept (in context)

Students had difficulty choosing two correct points from the diagram except those that chose the x and y intercepts. Candidates used the incorrect inequality sign when comparing two negative numbers. Verbose and circular arguments were presented to "justify" whether a wheelchair ramp had a safe gradient. Few candidates correctly rearranged the equation of a line. Examiners commented this question was weakly attempted by many candidates.

Question 8: χ^2

A number of candidates did not get the reasoning mark as to whether the null hypothesis should be "accepted"; frequently candidates wrote a vague statement such as "it is bigger" with no indication of what the word "it" refers to and lack of clarity of which numbers are being compared.

Question 9: currency conversion

Many candidates did not follow the directions in the question so answered to 2 decimal places rather than the nearest whole number and simply stated which was the best conversion rate with no supporting calculations.

Question 10: right angle trigonometry (in context)

Many candidates used sin instead of tan to find the missing length and did not write down both the rounded and unrounded answer when asked to give the answer to the nearest metre. The phrase "angle of elevation" is one of the new parts of the syllabus and many candidates were confused as to which angle to calculate. Candidates turned this into a much longer problem by using Pythagoras and then the cosine rule to find the angle of elevation. Inappropriate use of radians was rarely seen.

Question 11: exponential function (in context)

It is evident that many candidates use logarithms to solve exponential equations which is not in the current syllabus. Many candidates gave their answers with no working. Those that did show working used trial and error giving an answer to 1 significant figure.

Question 12: linear regression line (in context)

Candidates confused r and r^2. Many ignored that the variables of the problem were t and n so used x and y. A lack of working precluded those with a wrong linear regression line to obtain follow through marks.

Question 13: cubic function

Few candidates accurately drew the tangent to the curve. It was a rarity to see a candidate correctly identify the interval that the cubic function was increasing. Only the strongest candidates could find the equation of a tangent.

Question 14: normal distribution

It was evident a number of candidates had not studied the Normal distribution which is new to the syllabus. Many examiners commented this was the question most frequently left unanswered. Extremely few candidates sketched a normal distribution curve with the correct shaded region. The candidates that attempted the question did well.

Question 15: derivative of a function

Many students were unable to find the derivative of 4/x. The best candidates could use their GDC to find the co-ordinates of the local minimum.

Recommendations and guidance for the teaching of future candidates

Some candidates did not know how to use their calculator for example to find the local minimum of a cubic function. Teachers should explain how the GDC can be used to find features of a graph and to do the statistical applications such as the normal distribution. Mistakes in entering data into the calculator cannot be taken into account by the examiner so candidates should check carefully their entries to avoid a zero score on these types of questions.

Candidates should make sure they read the directions and give answers to the appropriate level of accuracy.

Candidates need guidance on how to justify answers using numerical evidence (as appropriate) and not just comments.

Candidates should be encouraged to show working as follow through marks cannot be awarded without this. For example when candidates were asked to find the time for an exponential function to reach a value many candidates gave the answer correct to only 1 significant figure with no working so scored zero out of a possible 3 marks.

All parts of the syllabus should be covered. Teachers should review the syllabus and be aware of how to teach new topics in the curriculum, such as Normal Distribution. Such topics may be missing from versions of textbooks written for the previous syllabus but there are plenty of other resources available.

Let candidates use the formula booklet during the teaching of the course so that they know what is in it and when to use it. Candidates should look at past papers to ensure they understand the way questions are asked. It is important to label question parts in the working to ensure answers are clearly communicated to the examiner.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 39	40 - 49	50 - 59	60 - 69	70 - 90

General comments

The paper seemed to be accessible. The majority of the candidates demonstrated good knowledge of the course material and ability to apply that knowledge to answer the exam questions.

The areas of the programme and examination which appeared difficult for the candidates

- In question 1, candidates faced some difficulties with conditional and combined probability.
- Candidates showed confusion while working with non right angled triangles. Many made used of the right angled triangles' concepts, namely, Pythagoras Theorem and SOHCAHTOA.
- In question 3, candidates could not always interpret the scales on the cumulative frequency curve.
- In question 4, answers were not given to the correct level of accuracy as instructed. Candidates mixed arithmetic progression formulae with that of geometric progression. Many candidates could not distinguish between the two progressions.
- In question 5, candidates did not succeed in sketching a graph with the help of the GDC. They could not show the asymptotic behaviour of the graph. Furthermore, many candidates did not show all the stages of work in "Show that" questions.
- In the last question of the paper, finding the value of "a" after substituting values and interpreting x appeared to have been difficult areas for the majority of candidates.

The areas of the programme and examination in which candidates appeared well prepared

- Most candidates were comfortable with the simple probability in question 1.
- In the second question, candidates demonstrated good knowledge of the sine and cosine rules. They made appropriate use of the rules and succeeded at the substitution of the correct values in their correct formulae.
- Candidates seemed to have been well prepared regarding the cumulative frequency table and interpreting the cumulative frequency graph.
- In question 4, candidates were well prepared in the use of the compound interest formula and the concepts of arithmetic progression.
- Many candidates were successful at the derivatives and volumes in question 5. Solving $\frac{\mathrm{d}S}{\mathrm{d}w} = 0$ algebraically or by using the GDC was not a problem to many candidates.
- In the last question, most candidates solved the quadratic equations successfully and demonstrated good knowledge of percentage volume.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: probability

Almost all candidates scored the 3 marks in part (a).

Part (b)(i) and (ii) were generally well attempted. Part (b)(iii) received a poor response from most of the candidates. Some candidates obtained values of more than 1 but did not realize that this value could not be correct.

Part (c) was well attempted.

In part (d), many candidates scored 1 mark for an attempt at 0.3 x 0.7.

Question 2: trigonometry

In part (a), most of the candidates were successful at showing 1220 m. Some candidates did not show their unrounded value 1216 and as such could not score both marks.

Part (b) was well attempted by almost all candidates. A few wrote 330 or 305 instead of 303.

In part (c), most of the candidates made use of the cosine rule and substituted the values correctly into the formula. Some did not apply the square root. Very few candidates used radian measures. Some of the candidates used the sine rule and some others considered the triangle to be a right angled one and made use of Pythagoras' theorem and/or SOHCAHTOA.

Part (d) was well attempted. Almost all students scored full marks at this part, their incorrect value at part (c) being followed through.

Part (e) was well attempted. Many candidates scored full marks or at least 2 marks for correctly substituting their values into the correct formula.

In part (f) very few candidates omitted the units. Most of the candidates used the correct formula; some lost 1 mark for an incorrect substitution. Those candidates made use of the incorrect sides and/or angle.

Question 3: statistics

Part (a) was well attempted.

Part (b) was well attempted except in some cases where the candidates rounded 12.5 to 13, which is incorrect in this question.

Part (c) was well attempted by most candidates. Many showed their workings. Some did it directly on their calculator.

Part (d) was answered well by candidates.

Part (e) was mostly well done. Some candidates misread the scales for 1cm representing 10 minutes. No candidate made use of the inequality sign in their answers.

Question 4: finance

Many candidates did not pay attention to the level of accuracy required in this question.

In part (a) candidate responses were very good.

Most candidates could work part (b) out without much difficulty. Many made use of the long list of 24 terms. Some candidates only found the 24^{th} term instead of the sum of the first 24 terms.

In part (c), many candidates obtained an incorrect ratio, *r*. They got 3 marks out of 5 if they had the correct method.

Many candidates obtained the correct answer to part (d) even if the previous parts were incorrect. Most candidates used the compound interest formula rather than the finance app on the GDC. Those who made use of the latter showed clearly all entries.

Many candidates did not score the mark in part (e) due to the fact that the previous parts were not attempted. As such, comparison between the options made no sense.

Part (f) was well answered by many candidates. Those who did not succeed at this part, failed at the substitution into the compound interest formula. Many candidates gave an answer of 8% and scored G0 if working were not shown.

Question 5: algebra

In part (a) most candidates gave their answer as 3000=20lw. Some used A for lw.

In part (b) most candidates gave good responses. As this is a “show that” question candidates had to show their stages of work clearly to score full marks.

Part (c) is a “show that” question was difficult to mark. Few candidates could clearly give an expression and then substitute in the expression.

In part (d) very few candidates scored full marks at this part. The most common errors were with the asymptotic behaviour and the smoothness of the graph.

Part (e) was well done. In some cases 4 was written as 40 and 300 written instead of –300. Some candidates left mistakes as they were simplifying the expression and differentiating at the same time. They performed a second derivative unconsciously or obtained 40+4=44 for the constant term in the derivative.

Parts (f) and (g) were well attempted.

Due to the nature of part (h), it was difficult to award for an answer of 109 or 110. Credit was given to candidates who gave at least 1 decimal place answer. This case was rather rare. Most candidates left the unrounded answer.

Question 6: functions and volumes

Most candidates did well in part (a).

In part (b) many candidates obtained $a = 0$.

In part (c) the most common mistake was the omission of the square on *x*.

Part (d) was well attempted.

In part (e) most of the candidates interpreted "*y*" correctly as the height. Very few candidates could interpret "*x*" in the context of this problem.

Part (f) was done well by many candidates.

In part (g)(i) many candidates multiplied 2.55 by 9. Part (g)(ii) was very well done by most candidates.

Recommendations and guidance for the teaching of future candidates

- Focus more on conditional and combined probability
- Candidates should be able to distinguish between right angled and non right angled triangles.
- Candidates found interpreting scales to be difficult. They must be better prepared at calculating the value of 1 mm (1 small square) in graphs.
- Students should be better prepared at the sum of an AP and a GP. Candidates must be encouraged to make use of the formulae instead of lists especially when many terms are concerned.
- Candidates should be encouraged to carefully read instructions and to give answers to the correct accuracy as required.
- Candidates must practice sketching unfamiliar graphs with the help of their calculator. They must be encouraged to use the given domain and to analyze their graph for key features. This will help them have better precision while sketching graphs.
- Candidates should be more familiar with asymptotes and the asymptotic behaviour of graphs.
- Candidates should be encouraged to show all stages of work; especially with "show that" questions.
- Make it clear to candidates that not showing workings may result in fewer marks.
- Candidates should be better prepared in answering questions in specific contexts. They must be prepared to apply concepts in different contexts.
- Candidates should reflect on the accuracy of their answers in the context of the problem.
- Candidates should be taught to use problem solving techniques in unfamiliar situations.