May 2013 subject reports

MATHEMATICAL STUDIES TZ2

(IB Africa, Europe & Middle East & IB Asia-Pacific)

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 17	18 – 32	33 – 44	45 – 57	58 – 70	71 – 83	84 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematical Studies papers.

Standard level Project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

The range and suitability of the work submitted

Most of the topics chosen this year were suitable for Math Studies projects. In some cases the topics were too descriptive and had little or no mathematical content. These should have been discouraged by the teacher.

As is the case every year, most candidates chose to write a statistical project. Other types of project using modelling, optimization, probability, trigonometry, sequences or finance were few and far between.

Most projects had a title. The clarity of the statements of task was variable as were the details of the plan. It is important for the candidate to write a clear plan explaining what they are going to do, what mathematical processes they are going to use in the project and give reasons as to why they are

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using these processes. This will help them to focus and will prevent them from including any irrelevant processes in their project.

A few candidates used statistical tests outside the syllabus and, even when the mathematics was accurate it appeared that they did not always fully understand the tests that they were using.

There were a number of very short and rushed projects that did not appear to satisfy the 20 hours of classwork and a similar amount of time for homework. There were also candidates who submitted a partial project of one or two pages in order to avoid being disqualified from obtaining a diploma.

Nearly all projects contained data which varied from a few pieces to hundreds of pieces. It should be noted that having a lot of data does not necessarily mean that it is quality data. However, there were several examples of projects with good quality data this session. Some candidates did not include their raw data. This makes it impossible for the moderator to know if it is quality data or to check that the tables are set up correctly or if the mathematical processes are accurate. Also, some candidates forgot to attach a copy of their questionnaire or survey. When using a random sample of data, the candidates should give an explanation of their method of choosing the "random" sample.

The simple mathematical processes were often done using technology without any explanation. The candidate should give an example of how to find a mean or show how to calculate the angles at the centre of a pie chart. A few candidates did not include any simple processes and jumped right into a chi-squared test and, as a result, their chi-squared test was counted as their first simple process and they did not score well in criterion C. The main errors in the sophisticated processes were, as always, in the chi-squared test (no null hypothesis stated, raw data or percentages instead of frequencies in the table of observed values, too many entries less than 5 in the table of expected values) and regression (drawing or calculating the regression line when the correlation coefficient was not moderate or strong). Also, there were a number of instances of chi-squared test being performed without any discussion of how the boundary limits for the cells were arrived at. There should be some discussion of whether the mean, median or some other value has been used and the reason for this choice discussed. Much use was made of technology with results occurring often without working, interpretation or justification. The teacher should encourage some calculations, as it is difficult for the moderator to verify that the candidates knew what they were doing. Most projects had at least one interpretation that was consistent with the analysis. Many candidates are now able to gain one mark for validity but very few are able to achieve full marks for this criterion. Projects generally had some structure but not always appropriate notation and terminology.

The guidance given to the candidates varied from school to school as did the quality of the teacher's comments on the 5/PJCS form. It is important for the teacher to write a comment against each of the assessment criteria, explaining why they awarded the marks, as this is helpful during the moderation process. Teachers should also write comments on the project and check the accuracy of the mathematics.

It is also very important for teachers to monitor the project during the various phases in order to avoid cases of plagiarism. There were more cases of plagiarism this year than ever before.

Candidate performance against each criterion

Criterion A:

Many candidates managed to gain two marks for this criterion. Those who did not usually did not have any clear plan to describe what they were going to do or their project did not have a title. Teachers should stress the importance of writing a clear statement of the task and a clear and detailed plan of how they are going to achieve this. This focuses the candidate and usually results in a project that is clear and follows a logical order. There are still some candidates who find this difficult to do in a clear and concise way. In many cases it is the result of choosing an unsuitable topic that should have been discouraged by the teacher. Most candidates explain how they are going to collect their data but do not describe the mathematical processes that they are going to use in the project. Those with clear statements of task and plan generally wrote more successful pieces of work.

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Criterion B:

Many candidates collected data that was appropriate for their project but this was not always sufficient to perform the mathematical processes laid out in their task. For others there was no problem with the quantity of the data but the quality was often questionable. Few candidates describe their sampling method and this is something that teachers could focus more on. Candidates who are using data from the internet or other secondary sources must also remember to identify the source in their bibliography. They also need to think about the relevance of this data for their project. All raw data must be included in the project in order for the moderator to check the accuracy of tables and mathematics. A sample of the questionnaire used must also be included along with the raw data so that the moderator can check the accuracy of any tables of values included in the project. Data that is too simple has a knock on effect for the whole project as it limits the mathematical processes that can be applied, the interpretations and the communication.

Criterion C:

The mathematics used in the project needs to be done in a relevant and meaningful manner. Some projects contained many mathematical calculations, some of which were meaningless for the actual project. In many of the projects the mathematics was done using technology. All mathematical processes using technology only are considered as simple processes. All processes such as mean, median, pie charts, chi-squared test, correlation and line of regression could all have been demonstrated by hand showing the moderator that the candidate knew what they were doing. Some candidates missed out any simple mathematical processes and only did a chi-squared test or line of regression. When no simple processes are present then the first sophisticated process is counted as simple. It is important for the candidate to realize this. As mentioned above there are still many errors in the chi-squared test and candidates are still drawing lines of regression on diagrams where there is little or no correlation present. This makes the process irrelevant and lowers the mark awarded for this criterion.

Criterion D:

The project flows better if partial conclusions are made after every mathematical process and then an overall conclusion given at the end. Most candidates managed to give at least one interpretation that was consistent with their analysis but fewer could produce thorough explanations of their calculations, often due to the fact that the project was too simple. Some candidates attempted to justify their results based on their own personal beliefs rather than the mathematics that they had performed. Teachers need to encourage candidates to ensure that their interpretations and/or conclusions are developed in a comprehensive way.

Criterion E:

Candidates are now commenting more on their data collection, their results and giving suggestions for extensions or improvement. Few are able to comment successfully on the validity of the mathematical processes that they have used throughout their project. Many candidates are now including their remarks about validity under a heading “Conclusions/Validity” as if they were two sides of the same coin. Candidates need help to understand that they need to choose which techniques to use and which not to use. Commenting on why they did or did not use a certain technique shows a good understanding of validity.

Criterion F:

The overall presentation of the projects was good. Most projects were word processed with tables and graphs that were clear to follow. Many projects had a reasonable structure but, due to errors in notation and terminology, only receive 1 mark for this criterion. The most common errors are: * for multiplication, ^ for “power of”, X^2 for the χ^2, E for “10 to the power of”, mixing up the correlation coefficient and the coefficient of determination.

Criterion G:

The majority of the teachers award this appropriately. Some schools abuse it and give full marks to all their candidates irrespective of the quality of the project.

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Recommendations and guidance for the teaching of future candidates

Teachers can help their candidates in many ways:

- Make sure that they read the Examiner's Report
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their candidates that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Give them examples of "good" projects so that they know what is expected of them.
- Peer assessment is a wonderful tool. Let the candidates moderate each other's projects.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the candidates the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage candidates to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Give candidates a second chance to correct errors.
- Emphasize the importance of meeting deadlines.
- Inform their candidates about sampling techniques.
- Remind their candidates to include all raw data either in an appendix or as part of the task.
- Show their candidates how to use equation editing software when word-processing.
- Remind them of the importance of including simple mathematical processes in their projects
- Check the calculations in each project.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded.

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Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 37	38 – 50	51 – 63	64 – 76	77 – 90

General comments

The paper appeared to be accessible with good syllabus coverage. The paper was of appropriate length and consisted of questions of varying difficulty. The majority of the candidates demonstrated a good knowledge and understanding of the syllabus. Many of them could select and use appropriate mathematical strategies and techniques while attempting the questions. It was pleasant to observe that candidates recognized and demonstrated an understanding of the practical applications of mathematics.

The areas of the programme and examination which appeared difficult for the candidates

The following areas appeared difficult to many candidates:

- Types of numbers.
- Stating the null hypothesis, finding the degree of freedom when the observed value table is not provided.
- Period, amplitude, range of a trigonometric function.
- Solving simultaneous equations (algebraically and using GDC)
- Probability – addition and multiplication of probabilities.
- Derivative of terms involving negative indices.
- Solving an equation involving negative indices.
- Solving exponential functions using the GDC.
- Compound Interest – rate compounded quarterly.

The areas of the programme and examination in which candidates appeared well prepared

The majority of candidates appeared to be well prepared in the following areas:

- Interpreting data from the cumulative frequency table.
- Writing the equation of a straight line.
- Finding the areas of shapes and required lengths.

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- Finding the median and mean of a set of data.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Number and Algebra

Many candidates did not pay attention to the word "exact value" in part (a). The different rounding offs required in part (b) were successfully done by the majority of candidates. In part (c), many candidates used their value obtained in part (a) or (b)(i) instead of the one from (b)(ii) to write in standard form, as instructed on the paper.

Question 2: Sets, Logic and Probability

Though most of the candidates did very well at implication in part (a), it was surprising to see that the concepts of disjunction and negation were not properly understood by many candidates. The second and third columns of the truth table were unsatisfactorily attempted by many candidates. In the last part of the question, not many candidates could give a clear explanation to why the statement was not a contradiction.

Question 3: Number and Algebra

Very few candidates scored full marks in this question. The main difficulty was with the set of rational numbers. Many candidates ticked $\sqrt{7}$ as a rational number. Surprisingly, many candidates could not get the set of real numbers correctly. Some numbers were missed out from the list.

Question 4: Statistics

This question was mostly well attempted by the majority of candidates. It was pleasing to see that candidates were correctly interpreting data from the cumulative frequency table. Part (b) was very well attempted too.

Question 5: Geometry and trigonometry

This question was fairly well attempted by some candidates. It was noted that very few candidates had their calculator on the radian mode. Those who had their calculator on radian mode could not figure out that their value to part (b) was producing an impossible answer. When the sine and cosine rules were correctly used, then the formulae were substituted properly by most of the candidates. It is to be highlighted, however, that many candidates had different answers, though correct to 2 significant figures, due to premature rounding off of their previous results.

Question 6: Statistics

This question was well attempted by many candidates. Even if the degree of freedom was incorrect, many candidates could obtain the critical value from the critical value table accordingly. They could also come to a conclusion consistent with their values from parts (b) and (c) by producing the correct comparison of the chi-squared calculated value and the chi-squared critical value. Unfortunately, many candidates are still using the word "correlated", "related" or "linked" to define the null hypothesis.

Question 7: Functions

In part (a) some candidates misinterpreted the term period with the coefficient of x in the equation $y = a\sin bx + c$. The amplitude was mostly written as –2 instead of the absolute value 2. Not many candidates could give the endpoints in the range. Many candidates understood that an inequality was expected in this part. Some candidates gave the range as a single number and wrote (3 – –1) = 4. In the last part of this question, several candidates managed to find the correct number of solutions. Some candidates did not pay attention to what was being asked and found the solutions instead.

Question 8: Geometry and Trigonometry

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Some candidates found it hard to rewrite the given equation of a line in the form $y = mx + c$. The coefficient of *x,* that is 2, was taken to be the gradient. Many candidates managed to find the gradient of the perpendicular line in part (b) by doing the negative of the reciprocal of their value from part (a). However, not many of them were successful at solving the simultaneous equations in part (d), whether, algebraically or by using their GDC. Candidates were rarely awarded method marks as no method was shown in the working box.

Question 9: Geometry and Trigonometry

Parts (a) to (c) were successfully attempted by the majority of the candidates. It was good to see that many candidates were including the correct units as part of their answers. Not many candidates could attempt part (d) correctly. This was mainly due to the fact that the correct formulae were not used.

Question 10: Sets, Logic and Probability

Part (a) was mostly well done. Very few candidates managed to do the next two parts correctly. In part (b), many candidates added the probabilities. In part (c), addition was used instead of multiplication and vice versa. Some candidates used probability with replacement.

Question 11: Introductory Differential Calculus

In part (a), few candidates could find the derivative of $\frac{-2}{x^2}$. Many candidates scored one mark for finding the derivative of *3x.* In part (b), among those who were successful at part (a), many found it hard to solve $x^{-3} = 8$. Instead of equating the derivative to 35, a common mistake in this part was the substitution of 35 in their derivative.

Question 12: Financial Mathematics

Part (a) was well done by many candidates. In part (b) partial marks were awarded to the candidates for showing their work. Many candidates scored at least one mark at (b) for showing their workings. Some candidates lost one mark for not giving their answer to the nearest dollar.

Question 13: Functions

In this question, not all candidates understood that $3^0 = 1$. Many used 3 instead of 1 in part (a). In part (b) 2 hours 30 minutes was used as 2.3 by some candidates. Candidates scored one mark for the correct rounding off of their value to the nearest hundred. In part (c), not many candidates could give the answer as 3.51 hrs. It was pleasant to see that workings were shown by many candidates. This allowed them to score partial marks in this part. The GDC was not always efficiently used to solve the correctly written equation.

Question 14: Statistics

In part (a), many candidates managed to find the value of *p*. In part (b), some candidates did not get the correct answer as they missed out one number while rearranging their list. Some candidates rounded off their decimal answer to the nearest whole number, e.g. 5.5 to 6. Part (c) was rather well attempted by many candidates.

Question 15: Financial Mathematics

The majority of candidates attempted the last question of the paper where many scored at least two marks for correctly substituting the compound interest formula. Not many candidates multiplied the number of years n by 4 and divided the rate r by 4. Answers were mostly given to the required number of decimal places. In part (b) many candidates managed to write a correct equation to find the rate, but very few were able to solve the equation.

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Recommendations and guidance for the teaching of future candidates

Future candidates are to be encouraged to avoid premature rounding off and hence accumulating errors.

Candidates must be encouraged to use exact values or values to at least 3 significant figures, unless otherwise indicated.

Candidates should use the GDC effectively to:

- Solve simultaneous equations; not necessarily in the form of $y = mx + c$.
- Sketch graphs to find point(s) of intersection.

Working must be clearly shown, for instance, a sketch of graphs from the GDC, algebraic manipulations and so on.

Teachers need to ensure that candidates understand:

- Use of OR and AND in Probability.
- Types of numbers.
- Compound interest, especially when the rate is compounded other than annually.
- Derivatives and equations involving functions containing terms with negative indices.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 32	33 – 44	45 – 54	55 – 64	65 – 74	75 – 90

General comments

The paper appeared to be accessible, and of appropriate length. The majority of the candidates demonstrated good knowledge of the course material and ability to apply that knowledge to answer the exam questions.

The areas of the programme and examination which appeared difficult for the candidates

The following tasks proved to be challenging for the candidates: Determining the probability of two events happening, identifying the correct axes and scale of the scatter diagram; using the GDC to find the correlation coefficient r, using the value of r to comment on the correlation, predicting changes in the graph of the line of best fit if new points are added to the data set, calculating total surface area and volume of a triangular prism, calculating an angle between a line and a plane, using AP and GP formulae in a context, determining that a given point is on the graph of a given function, calculating the derivative of a given function, using the derivative to draw inferences in the context of the question, writing down the equation of a line if the gradient and a point are known, and using the GDC to identify the coordinates of intersection points of two graphs.

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Candidates had the most difficulty with the trigonometry and the calculus questions. In the former, many candidates struggled with Question 3, part (a) which required the candidates to "show that" a statement is true. Many candidates had difficulties with using the derivative to identify whether a given point has a greater *y*-coordinate than the vertex of the parabola. A great majority of the candidates had difficulty with drawing conclusions and writing clear, succinct, and well-grounded justifications to support them. Some candidates lost all marks when they gave incorrect answers without showing their method, and lost the opportunity to gain the method marks. Many candidates lost a mark due to not giving the correct units.

The areas of the programme and examination in which candidates appeared well prepared

The majority of the candidates showed good time management skills and very few scripts had entire questions that were left unanswered. Good working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect. Many scripts were neatly presented although still not all candidates are organizing their working carefully.

Drawing and completing a Venn diagram, finding the mean of a set of numbers, drawing a line of best fit by eye, using the line of best fit to produce estimates, calculating area of a triangle, finding terms of AP and GP, and finding the sum of AP and GP, were well understood. The points plotted on the scatter diagram in Question 2 were mostly satisfactory. Most candidates were able to demonstrate good knowledge of the learned mathematical concepts and their applications.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Venn Diagram & Probabilities

Part (a) was answered well by most candidates. Parts (b) and (c) were mostly answered well. Many candidates received follow through marks from their Venn diagram. Some answers in part (b) were incorrectly given as fractions. Part (d) was the most problematic part of the question and few candidates gave correct answers.

Question 2: Scatter Diagram, Line of Best Fit, and Correlation

Many candidates reversed the axes in part (a), and had incorrect scales but the points were mostly plotted well. The coordinates of the mean point were mostly correctly calculated, and the point was well labelled. The line of best fit was often drawn not passing through point M and the correct *y*-intercept. The correlation coefficient was often correctly calculated, but in some cases was given as an incorrectly rounded number. The responses to the parts (g) and (h) of the question were particularly weak, and many candidates were not able to offer a satisfactory reason to support their conclusion. Many candidates were not able to clearly express their ideas.

Question 3: Geometry

In part (a) many candidates seemed to not know how to approach the question. Some did not give the unrounded and the rounded answer and lost the final mark. Part (b) was mostly answered well. Some candidates lost a mark in part (b), (c) or (d) for not using the correct units. Some candidates lost a mark for premature rounding in part (b) or (d). Many candidates treated the solid as a pyramid rather than a prism in part (e). Part (g) proved to be difficult for the candidates as well.

Question 4: Arithmetic & Geometric Sequences/Series

Parts (a) and (b) were mostly well answered. Many candidates were not able to give a satisfactory answer in part (c) but most of them were able to convert their answer given in seconds to minutes and gained at least one mark. Parts (d) and (e) were either answered well or incorrectly. Those candidates who gave incorrect answers were not able to recognize that they had to use the GP formulae. The responses to part (f) were often weak. Those candidates who set up the equation scored two marks but very few of them were able to reach the maximum marks.

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Question 5: Calculus

Part (a) was either answered well or poorly. Most candidates found the second term of the derivative in part (b) correctly, but the first term was often incorrect. Part (c) was rarely answered well – some candidates scored two marks for using their derivative to find the x-coordinate of the vertex but were not able to draw satisfactory conclusions about the location of point A in relation to the vertex. Part (d) had mixed responses. Part (e) was rarely answered correctly. Some candidates gave a correct equation, but were unable to convert it correctly to the specified form. Not many candidates were able to identify the coordinates of the required intersection point of the parabola and the line in part (f).

Recommendations and guidance for the teaching of future candidates

Show working – All relevant working should be shown in each question. Follow through marks can be then awarded where appropriate.

When showing work, label the part of the question you are answering. Proper labelling is necessary as much to help your quick review at the end of the exam as for the examiners when they review and mark your work.

Use GDC more effectively – Understand all the relevant functions and uses of the GDC. There is no need to explain how the GDC was used, i.e. which keys were pressed, etc. Candidates need to be encouraged to use their GDC throughout the entire course. Familiarity in using the calculators to graph unfamiliar functions and using it to solve equations is essential.

Check answers carefully – Candidates should be reminded to check their answers to ensure they are reasonable in the context of the question.

Pay attention to the required accuracy for the specific answers – Candidates should be reminded to give their answers to the accuracy required by a question, or to 3 significant figures otherwise. They must also know what penalties may be applied if the accuracy is not achieved or the specified units not used.

Know the command terms – Candidates should know all the command terms so that they know what action is required. They should also know the difference between “sketching a graph” and “drawing a graph,” and invest the appropriate efforts in the given task.

Learn to write succinct, clear, and well grounded justifications. It is important that candidates learn to communicate clearly.

Review past papers – Candidates should familiarize themselves with previous papers, their format, and key terms that are used.

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