May 2013 subject reports

MATHEMATICAL STUDIES TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 31	32 – 42	43 – 55	56 – 68	69 – 80	81 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2013 examination session the IB has produced time zone variants of Mathematical Studies papers.

Standard level Project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

The range and suitability of the work submitted

Most of the topics chosen this year were suitable for Math Studies projects. In some cases the topics were too descriptive and had little or no mathematical content. These should have been discouraged by the teacher.

As is the case every year, most candidates chose to write a statistical project. Other types of project using modelling, optimization, probability, trigonometry, sequences or finance were few and far between.

Most projects had a title. The clarity of the statements of task was variable as were the details of the plan. It is important for the candidate to write a clear plan explaining what they are going to do, what mathematical processes they are going to use in the project and give reasons as to why they are using these processes. This will help them to focus and will prevent them from including any irrelevant processes in their project.

A few candidates used statistical tests outside the syllabus and, even when the mathematics was accurate it appeared that they did not always fully understand the tests that they were using.

There were a number of very short and rushed projects that did not appear to satisfy the 20 hours of classwork and a similar amount of time for homework. There were also candidates who submitted a partial project of one or two pages in order to avoid being disqualified from obtaining a diploma.

Nearly all projects contained data which varied from a few pieces to hundreds of pieces. It should be noted that having a lot of data does not necessarily mean that it is quality data. However, there were several examples of projects with good quality data this session. Some candidates did not include their raw data. This makes it impossible for the moderator to know if it is quality data or to check that the tables are set up correctly or if the mathematical processes are accurate. Also, some candidates forgot to attach a copy of their questionnaire or survey. When using a random sample of data, the candidates should give an explanation of their method of choosing the “random” sample.

The simple mathematical processes were often done using technology without any explanation. The candidate should give an example of how to find a mean or show how to calculate the angles at the centre of a pie chart. A few candidates did not include any simple processes and jumped right into a chi-squared test and, as a result, their chi-squared test was counted as their first simple process and they did not score well in criterion C. The main errors in the sophisticated processes were, as always, in the chi-squared test (no null hypothesis stated, raw data or percentages instead of frequencies in the table of observed values, too many entries less than 5 in the table of expected values) and regression (drawing or calculating the regression line when the correlation coefficient was not moderate or strong). Also, there were a number of instances of chi-squared test being performed without any discussion of how the boundary limits for the cells were arrived at. There should be some discussion of whether the mean, median or some other value has been used and the reason for this choice discussed. Much use was made of technology with results occurring often without working, interpretation or justification. The teacher should encourage some calculations, as it is difficult for the moderator to verify that the candidates knew what they were doing. Most projects had at least one interpretation that was consistent with the analysis. Many candidates are now able to gain one mark for validity but very few are able to achieve full marks for this criterion. Projects generally had some structure but not always appropriate notation and terminology.

The guidance given to the candidates varied from school to school as did the quality of the teacher’s comments on the 5/PJCS form. It is important for the teacher to write a comment against each of the assessment criteria, explaining why they awarded the marks, as this is helpful during the moderation process. Teachers should also write comments on the project and check the accuracy of the mathematics.

It is also very important for teachers to monitor the project during the various phases in order to avoid cases of plagiarism. There were more cases of plagiarism this year than ever before.

Candidate performance against each criterion

Criterion A:

Many candidates managed to gain two marks for this criterion. Those who did not usually did not have any clear plan to describe what they were going to do or their project did not have a title. Teachers should stress the importance of writing a clear statement of the task and a clear and detailed plan of how they are going to achieve this. This focuses the candidate and usually results in a project that is clear and follows a logical order. There are still some candidates who find this difficult to do in a clear and concise way. In many cases it is the result of choosing an unsuitable topic that should have been discouraged by the teacher. Most candidates explain how they are going to collect their data but do not describe the mathematical processes that they are going to use in the project. Those with clear statements of task and plan generally wrote more successful pieces of work.

Criterion B:

Many candidates collected data that was appropriate for their project but this was not always sufficient to perform the mathematical processes laid out in their task. For others there was no problem with the quantity of the data but the quality was often questionable. Few candidates describe their sampling method and this is something that teachers could focus more on. Candidates who are using data from the internet or other secondary sources must also remember to identify the source in their bibliography. They also need to think about the relevance of this data for their project. All raw data must be included in the project in order for the moderator to check the accuracy of tables and mathematics. A sample of the questionnaire used must also be included along with the raw data so that the moderator can check the accuracy of any tables of values included in the project. Data that is too simple has a knock-on effect for the whole project as it limits the mathematical processes that can be applied, the interpretations and the communication.

Criterion C:

The mathematics used in the project needs to be done in a relevant and meaningful manner. Some projects contained many mathematical calculations, some of which were meaningless for the actual project. In many of the projects the mathematics was done using technology. All mathematical processes using technology only are considered as simple processes. All processes such as mean, median, pie charts, chi-squared test, correlation and line of regression could all have been demonstrated by hand showing the moderator that the candidate knew what they were doing. Some candidates missed out any simple mathematical processes and only did a chi-squared test or line of regression. When no simple processes are present then the first sophisticated process is counted as simple. It is important for the candidate to realize this. As mentioned above there are still many errors in the chi-squared test and candidates are still drawing lines of regression on diagrams where there is little or no correlation present. This makes the process irrelevant and lowers the mark awarded for this criterion.

Criterion D:

The project flows better if partial conclusions are made after every mathematical process and then an overall conclusion given at the end. Most candidates managed to give at least one interpretation that was consistent with their analysis but fewer could produce thorough explanations of their calculations, often due to the fact that the project was too simple. Some candidates attempted to justify their results based on their own personal beliefs rather than the mathematics that they had performed. Teachers need to encourage candidates to ensure that their interpretations and/or conclusions are developed in a comprehensive way.

Criterion E:

Candidates are now commenting more on their data collection, their results and giving suggestions for extensions or improvement. Few are able to comment successfully on the validity of the mathematical processes that they have used throughout their project. Many candidates are now including their remarks about validity under a heading "Conclusions/Validity" as if they were two sides of the same coin. Candidates need help to understand that they need to choose which techniques to use and which not to use. Commenting on why they did or did not use a certain technique shows a good understanding of validity.

Criterion F:

The overall presentation of the projects was good. Most projects were word processed with tables and graphs that were clear to follow. Many projects had a reasonable structure but, due to errors in notation and terminology, only receive 1 mark for this criterion. The most common errors are: * for multiplication, ^ for "power of", X^2 for the χ^2, E for "10 to the power of", mixing up the correlation coefficient and the coefficient of determination.

Criterion G:

The majority of the teachers award this appropriately. Some schools abuse it and give full marks to all their candidates irrespective of the quality of the project.

Recommendations and guidance for the teaching of future candidates

Teachers can help their candidates in many ways:

- Make sure that they read the Examiner’s Report
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their candidates that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Give them examples of "good" projects so that they know what is expected of them.
- Peer assessment is a wonderful tool. Let the candidates moderate each other’s projects.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the candidates the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage candidates to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Give candidates a second chance to correct errors.
- Emphasize the importance of meeting deadlines.
- Inform their candidates about sampling techniques.
- Remind their candidates to include all raw data either in an appendix or as part of the task.
- Show their candidates how to use equation editing software when word-processing.
- Remind them of the importance of including simple mathematical processes in their projects.
- Check the calculations in each project.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 26	27 – 35	36 – 48	49 – 60	61 – 73	74 – 90

The areas of the programme and examination which appeared difficult for the candidates

The following areas were less well answered: significant figures, Venn diagrams (especially number of elements), interpretation of cumulative frequency curves, commenting on skewed data, some confused arithmetic and geometric sequences, manipulating algebraic equations, and sine curve.

The areas of the programme and examination in which candidates appeared well prepared

Almost all candidates were able to correctly add the probabilities to a tree diagram, write a number in standard form, substitute values into the formulae in the information booklet, convert currencies and differentiate a function.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Significant Figures and $\boldsymbol{a} \times \mathbf{10}^{\boldsymbol{k}}$ Form

Many candidates calculated $\frac{17x^2}{a} - b$ instead of $\frac{17x^2}{a-b}$ on their calculators; however they were able to get follow through points. It is important that candidates learn how to correctly input expressions into their calculators. Although the question explicitly stated in bold to use the answer to **part(b)(ii)** many candidates used their answer to part (a) for part (c). The general notes about rounding in the mark scheme are over-ruled if the question has explicit directions such as in this question.

Question 2 Venn Diagrams

Many candidates were unable to write down correctly the universal set which was integers between 2 and 10. Some candidates did not read the direction “on the Venn diagram” so complained of lack of space for their answer. It is important candidates read the directions carefully. Many candidates listed the elements of the intersection rather than answering the question to specify the number of elements. The empty set for $(A \cup B)'$ was awarded a maximum of 2 marks as this has simplified the problem.

Question 3 Logic

Some candidates found the phrase “Yuiko is studying French but not Chinese” confusing as they did not realize in this context the word “but” means “and”. Alternative but correct logic notation was accepted.

Question 4 Cumulative Frequency

The reading of values from a cumulative frequency was difficult for candidates and a notable number of candidates left this question unanswered or scored zero.

Question 5 Probability

Probability tree and calculations from this were well done and were straight forward.

Question 6 Truth Table

This was provocative in the G2 and the comments indicate that candidates found the wording confusing. Candidates were able to write in words the compound proposition $\neg p \vee q$ and following from their truth table the candidates could state if this was true or false. In part (c) many candidates either stated the correct answer "true" or stated an answer consistent with their truth table and received follow-through marks. Candidates had difficulty writing down a value of x for which $\neg p \vee q$ is false.

Question 7 Geometric Sequence

The weakest candidates erroneously used an arithmetic sequence rather than a geometric sequence as specified in the question.

Question 8 Box and Whisker

Many candidates omitted the "kg" units that were required for the median weight. It is not only area and volume answers where marks may be lost for either missing or incorrect units. Candidates confused IQR with range. Only the very strongest candidates were able to deduce from a box and whisker plot that the data was asymmetric (with a positive skew) hence the mean was greater than the median. This was one of two reasoning marks in the paper and only the very strongest candidates wrote down a correct reason.

Question 9 Quadratic Function

Many candidates did not see the connection between the *x*-intercepts and the factored form of a quadratic function. The syllabus explicitly sates that the graphs of quadratics should be linked to solutions of quadratic equations by factorizing and vice versa. This was one of the most challenging questions for candidates.

Question 10 Coordinate Geometry

There were multiple acceptable reasons why the line did not pass through a given point (including numerically substituting values in the equation; drawing a graph or algebraically finding the *x*-intercept of the line). This was one of two reasoning marks in the paper.

Question 11 Currency Exchange

Marks were awarded in part (a) for multiplication by 0.024 in part (b) for division by 1.2945 and in part (c) for multiplication by 19495. Candidates did not follow specified levels of accuracy. Candidates were able to answer later parts of the question even if they did not answer the first parts correctly.

Question 12 Arithmetic Sequence

Some candidates confused geometric sequence with arithmetic sequence. Candidates found the algebraic manipulations difficult so scores on this question were weak.

Question 13 Sine Curve

The question stated that the measurements were in degrees however a few used 2π instead of 360°. If the calculator was set in radians the candidates would have made no difference to their answers as no trigonometric values were evaluated. Many candidates subtracted the endpoints and hence erroneously wrote 12 hours rather than stating the time interval when the depth of the water in the reservoir was decreasing. This was one of the most challenging questions for candidates.

Question 14 Simple and Compound Interest

Although this question was late in the paper there were a number of candidates who scored full marks. Others were able to do simple interest but not compound interest or vice versa. However there were a large number that scored zero.

Question 15 Differential Calculus

Many candidates could find the derivative of the cubic function and find the value of the derivative at $x = 0$. For part (c) many candidates calculated the value of the function rather than the derivative at $x = -2$. However only the best realized that the derivative is zero at the maximum and so calculated the value of a.

Recommendations and guidance for the teaching of future candidates

- Candidates need to understand mathematical vocabulary. TZ1 is sat predominantly by American candidates and many did not realize that “gradient” is slope.
- Some candidates did not follow the directions in the question; even though **bold** was used for emphasis. Candidates should write down the unrounded answer then round according to the directions of the question.
- “Plug numbers into calculator” or “from GDC” is neither a reason nor a mathematical process. Teachers need to show candidates how to record the step(s) leading to their answer even if they used a calculator to find their solution.
- It is important that working is shown so that follow through marks can be earned if an earlier part is wrong.
- Some candidates wrote down a correct equation followed by a wrong answer through inability to correctly enter what they had written down into their calculator.
- It is important to cover the entire syllabus; there were candidates who scored full marks or close to it for at least 10 questions and then left two or three questions unanswered.
- Past papers are a good source of practice questions. Working on these questions in timed test conditions helps candidates learn to manage their time in examinations.
- Some candidates crossed out work which was not replaced – so threw away marks.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 30	31 – 42	43 – 52	53 – 62	63 – 72	73 – 90

General comments

The majority of candidates attempted all the questions, though there were a number of trivial attempts at questions 1 and 4; the chi-squared test and the calculus. It may be that time was an issue for a small minority of candidates, however, it seems more likely that some candidates were not prepared specifically for this syllabus. The wide range of marks indicated that the better candidates were able to display their knowledge and skills over the entire paper. The examination was deemed to be an appropriate test of the syllabus by the majority of teachers submitting G2 forms and the clarity of wording seems to have been acceptable to the majority.

It was also commented that the syllabus coverage was good once taken in concert with Paper 1.

As in ever the case, a number of candidates lost marks in the "show" questions. It is again reiterated that when candidates are required to reach a given answer that is written to a specified accuracy, they must first write down the value they obtain, correct to a higher degree of accuracy, and then write down the given value so that these can be seen to be the same.

In the question asking for angles, the use of radians continues to decline; however, the loss of the correlation coefficient due to resetting the TI GDC was again evident.

The areas of the programme and examination which appeared difficult for the candidates

- Drawing a graph to the correct scale.
- Comparing appropriate values to make a decision in a chi-squared test for independence.
- Representing information on a Venn diagram.
- Calculating conditional probability.
- Finding the equation of the asymptote.
- Expressing the regression line with appropriate variables.
- Calculating the percentage error.
- Finding and using the gradient function.
- Finding the range.
- Finding the tangent line for the correct point on the curve.
- Distinguishing between draw/sketch, find/calculate/show/write down.
- Expressing answers to an appropriate degree of accuracy.

The areas of the programme and examination in which candidates appeared well prepared

- Data was accurately plotted on their graph.
- Understanding of coordinate geometry.
- Application of Pythagoras' theorem.

- Calculation of the area of a triangle and rectangle.
- Finding the chi-squared calculated statistic and the *p*-value from the GDC.
- Units of measurement being shown.
- Sine rule to solve non right angle triangles.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Chi-Squared Test

The great majority of candidates found this question to be a good start to the paper. The common errors were (1) incorrect terminology in the null hypothesis, (2) use of the 5% level, (3) an inability to find the expected value by hand, (4) comparison of incorrect values. Note, candidates will **never** be asked to calculate the chi-squared statistic other than from the GDC.

Question 2: Sets and Probability

This question divided the candidates into two parts: those who knew how to interpret the information in a manner the led to a consistent Venn diagram and those who did not. The use of the word "only" is crucial in this regard.

Follow through to the probability part of the question was contingent on the use of the given $n(E) = 22$; given information should be used in subsequent parts. As ever, conditional probability proves a trial for many.

It is recommended that candidates write probabilities as unsimplified fractions as this increase their chances of gaining follow through from previous parts.

Question 3: Cooling Curve and Correlation

Almost all candidates were able to score on the first parts of this question; errors occurring only when insufficient care was taken in reading what the question was asking for. The graph was usually well drawn, other than for those who have no idea what centimetres are.

The majority were able to determine the simultaneous equations, if only in unsimplified form; there was less success in solving these – though this is easily done via the GDC (the preferred approach) and the equation of the asymptote proved a discriminating task. The final parts, involving correlation and regression were largely independent of the previous parts and were accessible to most. Hopefully, contrasting the large percentage error with the value of the correlation coefficient will be valuable in class discussions. Given the many scripts that gave the value of the coefficient of determination as that of r, it seems better that the former is simply not taught.

Question 4: Calculus

Most candidates were able to evaluate the function and find the derivative for x+6 but the term with the negative index was problematic. The few candidates who equated their derivative to zero at the local minimum point progressed well and showed a thorough understanding of the differential calculus. Many did not attain full marks for the range of the function, either confusing this with the statistical concept of range or using the *y*-coordinate at B. Most were able to find the gradient and midpoint of the straight line passing through A and B. The final parts were also challenging for the majority: many had difficulty finding the gradient of the tangent L, instead using the slope formula for a straight line; the most common error in part (i) was to substitute in the coordinates of midpoint M rather than the point on the curve. Greater insight into the problem would have come from using the given sketch of the curve and annotating it; it seems that many candidates do not link the algebraic nature of the differential calculus with the curve in question.

Question 5: Trigonometry and Area

Most candidates were able to recognize sine rule, substitute correctly and reach the required result.

The use of Pythagoras' theorem was also successful, the major source of error being the lack of unrounded and rounded answers in part (d). Part (e) was less well answered, due in part to the triangle being in three dimensions. However, all three sides had either been asked for in previous parts or given and all that was required was a sketch of a triangle with the vertices labelled; such a diagram was never on any script and this technique should be encouraged.

Again, most candidates used the appropriate area formula – however, some did not read the question with the attention it required and found the area of three rectangles – one of which being the stated "concrete base".

Recommendations and guidance for the teaching of future candidates

- Use SI units – if only cm and mm for graphs.
- Time management – a mark a minute is the guide – and ensure that all questions are attempted.
- Cover the whole syllabus; it will all be examined – if not in Paper 2 then in Paper 1.
- Though the candidates appeared to take care in the accuracy of their calculations and answers, this is a point that requires constant reminder. Premature rounding is likely to be an issue for multi part questions. Candidates should show and use unrounded answers as much as possible.
- Show units.
- Read each question carefully.
- Understand the commands such as *Find*, *Show that*, *Sketch*, *Draw*, and *Calculate*.
- Encourage candidates to show all calculations and display the steps they make. The 'Show' command requires candidates to state both the unrounded and final answers.
- Candidates need to be competent in using mathematical techniques as per the course guidelines. It is possible to be overly dependent on the GDC and so the candidates do not give adequate attention to some skills. By the same token they need to be aware of how to utilize the advanced features of the GDC.
- Candidates should be conversant with appropriate terminology for each area of the course; this is important when being asked to explain their reasoning.
- Where possible use diagrams and sketches to illustrate the information.
- Start each question on a new page.
- A graph should be sketched accurately and axes labelled and scaled; points should be joined by a smooth curve.