MATHEMATICAL STUDIES TZ1

(IB Latin America & IB North America)

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-16	17-30	31-44	45-57	58-71	72-83	84-100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2012 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level, to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-8	9-11	12-14	15-16	17-20

Range and suitability of work submitted

The majority of the projects chosen this session were suitable and used statistics as the main mathematical process. Most of the statistical projects used Pearson's correlation coefficient or the chi-squared test. In many cases there were serious flaws in the processes. It was a delight to see projects from areas other than statistics and teachers are encouraged to promote this.

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Many students this session failed to include their questionnaire or raw data in the project, making it impossible for them to score highly in Criterion B or C.

In projects where the student collected their own data, often they did not describe the data collection process in sufficient detail to allow for the assessment of the quality of the data.

Many students only included one sophisticated process in their project and omitted all simple mathematical processes. In this case the first sophisticated process is considered "simple". A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering their test invalid. Students also confused correlation and independence. What is more surprising is the fact that teachers are not highlighting these mistakes. Either they are not checking the mathematics or they themselves do not understand the test.

It must be emphasized that answers given from using technology only are considered simple. In order to gain high marks in Criterion C a candidate must show the mathematical processes by hand.

Teachers need to be aware that if there is incorrect notation or terminology in the project then no more than 1 mark can be awarded for Criterion F.

Most candidates are now scoring 1 mark for Validity but very few gain full marks for this criterion.

The project should reflect the 20 hours allocated for school work plus approximately the same amount of time outside of the classroom. This session many projects looked like homework exercises rather than a substantial piece of work.

Some school are sending copies of the projects to the moderator instead of the original. This makes it very difficult for the moderator to check any bar charts or pie charts that are now "shades of grey". The original, in colour, should always be sent to the moderator.

As always there were some candidates who produced excellent projects that scored well in almost every assessment criterion.

The teachers must put a comment against every criterion on the 5/PJCS forms explaining why they awarded the marks that they gave the student. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against the criteria

A. Most candidates mentioned their task but the plans varied from thorough to non-existent. If the plan is clearly stated then the rest of the project should flow from it. It is worthwhile spending time on the stated plan. A few projects did not have a title making it impossible to get more than 1 mark for this criterion.

B. Many candidates fail to achieve full marks in this criterion for various reasons: the data was not set up for use, the quality of the data was questionable, the quantity was limited, the raw data was omitted or the source of the data was omitted. If the raw data is not present then the moderator cannot check the accuracy of the mathematical processes used. Data that is too simple invariably results in limiting the mathematical analysis that the candidate can perform.

C. Many projects only contained 1 sophisticated process. This is then counted as the first simple process and is marked accordingly. Many candidates seem to be unaware of the conditions that make the chi-squared test invalid: not using frequencies for the entries in the contingency table, expected values less than 1, more than 20% of expected values between 1 and 5. Candidates must also be aware that finding the equation of the regression line when the correlation coefficient is weak is not a relevant process and will have the effect of reducing their marks. If technology only is used to arrive at results then this is considered a simple process.

D. If a project is simple then it is not possible to produce a detailed discussion of results. Most candidates however did score 2 marks for this criterion. The better candidates could discuss their results thoroughly and received full marks.

E. Candidates are now attempting to discuss validity and many received 1 mark for this criterion. However, it seems as if many of the candidates are not really aware of what “validity” actually is and it would be beneficial if the teacher spent some time with their students explaining what is required for this.

F. The majority of the projects had some structure with candidates recording their actions at each stage. However, many candidates lost marks due to errors in either notation or terminology. Some candidates do not seem to be aware that calculator/computer notation is not correct mathematical notation.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Give them examples of "good" projects so that they know what is expected of them.
- Peer assessment is a wonderful tool. Let the students moderate each other’s projects.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.

- Teach the students the significance and limitations of statistical techniques.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage students to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Emphasise the importance of meeting deadlines.
- Inform their students about sampling techniques.
- Remind them to include all raw data either in an appendix or as part of the task.
- Show their students how to use Equation editor or Math Type.
- Remind them of the importance of including simple mathematical processes in their projects.
- Check the calculations in each project.
- Send the **original** work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-40	41-52	53-65	66-77	78-90

General Comments

The comments on the G2 forms were most appreciative of the syllabus coverage and of the level of difficulty of the exam paper. This was reflected in the candidates' responses where generally most questions were well answered with all necessary working shown enabling follow through marks to be awarded in latter parts of the questions.

The areas of the programme and examination which appeared difficult for candidates

- Equation of axis of symmetry and writing down a required coordinate rather than an ordered pair of coordinates.
- Interpreting percentage of data less/greater than a given quartile on a box and whiskers diagram.
- Understanding what is meant by the inverse of a logical proposition.
- Evaluating constants in periodic functions.
- Identifying the correct side or angle of a figure required from the demand of the question.
- Correct reasoning to justify reliability of using an estimate from a line of regression.
- Conditional probability.
- Interpreting mapping diagrams.
- Correct use of a compound interest formula.

The areas of the programme and examination in which candidates appeared well prepared

- Averages.
- Coordinate geometry.
- Representation of numbers in standard form.
- Interquartile range.
- Currency conversion
- Equation of a line in 2 dimensions.
- Use of the GDC to find the equation of a line of regression.
- Probability of combined events.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Measures of central tendency

In part (a), the majority of candidates were able to identify the correct value for x. In part (b), many candidates seemed to think that the median was the same as the value of y. Consequently the value 5 proved to be a popular, but incorrect, answer. The majority of candidates wrote down the correct value of 7 and went on to give the required answer of 16 in part (c). A cautionary note here though: A common set of responses to this question was 1, 5 and 18. Without working this earned 1 mark, with working (in part (c)) this earned 4 marks.

Question 2: Coordinates in two dimensions

(a) Despite some good answers in this part of the question, sign errors in setting up one or both of the equations meant that marks were lost by some candidates. This error was particularly prevalent in finding the value of p and the equation $\frac{(p+3)}{2}=1$ was seen quite often. In part (b), there was a requirement for a correct substitution of their coordinates for A and B into the correct formula for Pythagoras and, while $(2+4)^2$ was often seen, sign errors in the second component of $(-3-5)^2$ proved to be the downfall of a significant number of candidates. As a consequence, the final two marks were lost. Given these errors, there were still a significant number of full mark responses to this question.

Question 3: Standard form

(a) The large numbers given in the stem of this question led to some candidates being a factor of 10 out in their answer to part (a) and therefore losing at least the A mark. Errors were compounded in this first part of the question with some candidates dividing by 48000^{-1} which effectively meant multiplying by 48000. Much good work, however, was seen in the remaining two parts of the question with candidates well able to show a correct standard form from their figures.

Question 4: The graph of a quadratic function

(a) Identifying '2' and leaving this as the answer was not sufficient for any marks in this part of the question as was simply leaving the equation $2+\frac{-b}{2a}$. In part (b) whilst much good work was seen by some candidates in sketching the correct curve, others failed to recognise the symmetry, joined the given points with straight lines or simply drew curved segments which were far from smooth. Part (c) required, for one mark, the writing down of the *x*-coordinate of the point D. A significant number of candidates, including very able candidates, lost this mark by writing down $(3,0)$.

Question 5: Box and whisker diagram

Parts (a) and (b) proved to be very well done with many correct answers seen. On a few scripts however, candidates who seemed unsure of the correct average, wrote down, average, mean or even medium. Part (c) was generally well done with many candidates correctly identifying Q1 and Q3 and many correct answers of 5 were seen. 75% proved to be an elusive answer on many scripts for part (d) as a significant number of candidates did not seem to understand the meaning of quartiles. Indeed, a popular, but erroneous answer seen was 57.1% which was arrived at from the calculation $\frac{14-6}{14}\times 100$.

Question 6: Logic

Although a few candidates did not seem to understand the meaning of the $\Rightarrow$ symbol, many scored a minimum of two marks on the first two parts of the question. Indeed, many correct statements were seen in part (a). Many candidates however confused converse with inverse in part (b) resulting in the incorrect statement '*if the sum of* x *and* y *are both even then the numbers* x *and* y *are both even*' appearing on many scripts earning *(M1)(A0).* Despite this incorrect compound statement, many candidates recovered with correct reasoning in part (c) from their correct (or incorrect) statement in part (b). Candidate's responses to part (c) of the question should have been given in the context of the question set and those that simply inferred their answer from truth tables only, earned no marks.

Question 7: Graph of a cosine function

In part (a), the question required the *time* at which the water reaches its maximum temperature. As a consequence, an answer of 12 did not earn the mark here. Part (b) was done well and many candidates were able to correctly identify the critical values of 6 and 18 in part (c). However, the 2nd mark in part (c) proved to be elusive to many candidates as either $\leq$ symbol was used or a variable x instead of t was seen. In part (d), candidates who recognised that to obtain *b* they simply had to divide the period, 360, by the time taken for this period and to arrive at an answer of 15 earned both marks. Candidates however who tried to form and solve an equation involving the given cosine function were far less successful and much incorrect working was seen.

Question 8: Currency conversions

A lot of good work was seen in this question with many completely correct solutions. The two marks which were seen to be lost more often than others were the answer to part (b) where \$1.84 was seen rather than the required answer of 23 MXN, and the final mark in part (c) where, in many cases, the answer was left as \$ 182.16.

Question 9: Equation of a line in two dimensions

In part (a), the word '*explain*' required more than simply stating that '*I put the coordinates into my GDC and it did not work*'. A written statement, showing the substitution of one or both of the coordinates leading to an inequality was required for this first mark. It was pleasing to see that many scripts showed correct methodology for calculating the gradient of *L* and the gradient of a line perpendicular to *L*. The correct equation of the line perpendicular to *L* passing through P proved to be more elusive as poor arithmetic spoilt what could have been excellent work. In particular $-5=\frac{3}{2}\times 6+c$ leading to $c=(\pm)4$ proved to be a popular but erroneous calculation.

Question 10: Cosine and sine rule

In part (a), candidates seemed to be well drilled in the use of the cosine rule and AC = 14 m proved to be a popular, and correct, answer seen. The most popular incorrect answer seemed to be 11.7 m. This seems to have been arrived at by simple Pythagoras on triangle DCA – clearly, a totally incorrect method. Not as many candidates then went on to find the required angle in part (b). Some simply continued with using triangle DCA and found angle DCA. Indeed, some candidates even found angle DCB rather than the required angle ACB. In most cases, correct or otherwise, the sine rule was used and credit was awarded for this process.

Question 11: Use of GDC to find the equation of a regression line

This question was an opportunity for candidates to show effective use of the GDC and many correct answers were seen in parts (a) and (b). Part (c) was unusual in that questions of this nature, in the past, have focused on values outside of the range of given values of *x*. For correct reasoning, candidates were required to identify that 17 was in the range of values given for *x* in the table. Identifying that 17 was between 15 and 20 minutes was sufficient whereas identifying that 173 was between 125 and 200 was clearly not sufficient. Alternative reasons which focused on the correlation coefficient being either strong positive or close to one were seen and were accepted.

Question 12: Probability

It was pleasing to see many correct answers in parts (a) and (b) with many writing their answer to part (b) in the context of the question and writing down a percentage. Conditional probability is not an easy topic for candidates to understand and many simply wrote down $0.6 \times 0.25 = 0.15(15\%)$ for part (c).

Question 13: Sine rule and area of a triangle

Whilst using the sine rule to find an angle was tested in question 10, here the sine rule was required to be used to find a length. Many scripts showed the correct value of 6.22 m, but a significant number of candidates calculated AB instead of AC and the answer of 11.7 m proved to be a popular, but erroneous, answer. Some candidates turned the problem into two right angled triangles by dropping the perpendicular from C to the line AB. Much working was then required to find AC and again, some of these candidates simply determined the length of AB. Despite a significant number of candidates identifying the incorrect length in part (a), many of these recovered in part (b) to use their value correctly in the formula for the area of a triangle and many correct calculations were seen. Unfortunately, this good work was spoilt as some candidates either missed the units or gave the incorrect units in their final answer and, as a consequence, lost the last mark.

Question 14: Mapping diagram and equation solving

Candidates both understood how to interpret a mapping diagram correctly and did very well on this question or the question was very poorly answered or not answered at all. Writing down two simultaneous equations in part (a) proved to be elusive to many and this prevented further work on this question. Candidates who were able to find values of *p* and *q* (correct or otherwise) invariably made a good attempt at finding the value of *s* in part (c).

Question 15: Simple and compound interest

In part (a) there was much confusion between whether to use 25 000 or 12 500 and/or an incorrect use of the simple interest formula $\left(\frac{12500 \times 0.05 \times 18}{100}\right)$ led to many incorrect answers in this part of the question. Indeed, many incorrect interim or even final answers of 22 500 EUR or 112.50 EUR were seen. Of those who did interpret the question correctly and apply a correct use of the simple interest formula, a significant number failed to add on 12 500 so earned, at most, one mark for this part of the question. In part (b) a mark was awarded for substitution of **any** values into a compound interest formula. This seemed to be as far as the majority of candidates were able to go and few scripts gave the required answer of 12183.39 EUR.

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- Clearly identify in the working box by using the appropriate labels (a), (b)...., the part of the question being answered and writing their final answer on the answer line. Candidates should be encouraged **not** to write their answers against the question demand as this may lead to their answer not being marked.
- Give all answers to the required degree of accuracy identified in the demand of the question. Where there is no demand for a level of accuracy within a question, give an unrounded answer (where possible) to more than 3 figures and a final answer to 3 figures.
- Critically examine their answers to see whether or not they are sensible in the context of the problem set.
- Show all working to enable method marks to be obtained if answers are incorrect.
- Not cross out their work unless it is to be replaced – crossed out working earns no marks at all.
- Ensure that they are fully conversant with the formulae which appear in the information booklet and where exactly these formulae are to be found in the booklet prior to the examination.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-41	42-52	53-63	64-74	75-90

General Comments

There were many comments on the G2 forms about a lack of time to complete the paper, and perhaps there was a greater number of candidates who failed to complete the paper than in previous years; however, the majority attempted all five questions, though there were a number of trivial attempts at questions 2 (the chi-squared test) and 5 (the calculus). If one considers question 5, it may be that time was a problem for some candidates but the continued lack of knowledge of the chi-squared test of some candidates is disturbing, since this is one of the more accessible parts of the syllabus. The examination was felt to be an appropriate test of the syllabus by the majority of teachers who submitted G2 forms.

With the onset of online marking, the importance of the level of accuracy of candidates' answers assumes more importance, since **level of accuracy is assessed in each question, independent of all others**. It is important that the guidelines below are followed:

- Answers that are exact should be given in full (In Q3, for example, answers as multiples of π are acceptable).
- Non-exact answers should be corrected to (at least) three significant figures. The use of 3.14 as an approximation for π should be actively discouraged – candidates must use the π button on the calculator.
- Where an answer to a question part is required in a subsequent part, the full calculator display answer should be used. It is a simple matter to "cut and paste" this answer in the appropriate place.

Further, again due to the onset of online marking, it is imperative that candidates do not write on the question paper; question papers are not scanned for marking.

As ever, candidates lost marks in the "show that" parts of the questions. When candidates are required to reach a given answer that is written to a specified accuracy, they must write down the required value to a higher degree of accuracy (unrounded value) and the given answer. In such cases, such as Q3 (b), an exact answer in terms of π is not acceptable.

Pleasingly, the use of radians in questions asking for angles continues its downwards trend; however, the inability of many candidates to discriminate between cm and mm for graph drawing is disturbing; this is an international examination and SI units are an integral part for the course.

The areas of the programme and examination which appeared difficult for candidates

- Curve sketching.
- Applications of the differential calculus.
- Showing how to obtain expected values in a chi-squared test.
- Setting up H_0 and obtaining the result a chi-squared test.
- Applications of APs.
- Compound probability.

The areas of the programme and examination in which candidates appeared well prepared

- Venn diagrams.

- Chi-squared test on the GDC.
- Standard use of AP/GP formulas.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Set Theory and Percentage Error

This question was accessible to the great majority of candidates. The common errors were:

- the lack of a bounding rectangle in (a);
- the lack of subtraction for the entries in the disjoint regions of the type $A' \cap B' \cap C$ and the subsequent total exceeding 100%;
- the **incorrect** interpretation of “either ...or” as “exclusive or”. It is of the utmost importance to note that the ambiguity of the “or” statement will be removed and “exclusive or” signalled by the phrase “either ...or....**but not both**”. Otherwise, “inclusive or” must always be assumed.

A number of candidates were unable to interpret the percentage error question correctly and scored 0/4. This was somewhat disappointing.

Question 2: Chi-squared Test and Probability

This question was successfully attempted by the great majority. However, the test is for the mathematical independence of the two variables; it does not address “correlation” or whether there is “no relation” between them. Further, the result of the test should be determined by the comparison of the **numerical values** of either the chi-squared calculated and critical values or the associated *p*-value and the significance level of the test. The creeping use of *k* as the critical value is the notation used in one text book; it is **not** standard notation and its use is not accepted. Comments were made on the G2 forms as to whether the the null hypothesis should be “accepted” or not rejected; both forms are acceptable.

In the compound probability questions, the lack of an explicit tree diagram determined that many candidates were not able to proceed. Determining an appropriate technique is a skill that should be taught.

Question 3: Areas and Volumes

This question was either very well done – by the majority – or very poorly (but not both). Many incomplete attempts were seen. This would perhaps indicate a lack of preparation in this area of the syllabus from some centres, since it was that the formulas for cones were not well understood. Further, the idea of “total surface area” was a mystery to many – a slavish reliance of formulas, irrespective of context, led to many errors and a consequent loss of marks.

The invariance of volume for solids and liquids that provided the link in this question was not understood by many, but was felt to be an appropriate subject for an examination.

Question 4: Sequences and Series

Part A: Arithmetic

The contextual nature of this question posed problems for many, though there were many fine attempts. Failure to discriminate between the sequence and series formulas was the cause of the most errors. The final part saw many able to substitute into the formula for the series, but then unable to continue. The use of the GDC in such situations is encouraged; either by graphing each side of the equation and drawing the resultant sketch or by the solver function.

Part B: Geometric

The early straightforward parts were accessible to the majority. The context caused the problems with many choosing the incorrect value of n when using the formulas. Weaker candidates were more successful via counting. The context again proved challenging in the final part, with the incorrect time being determined from the correct value of n. Here, as in Part A, the use of the GDC by graphing each side of the equation is encouraged; however, if teachers feel that such questions require the use (and teaching) of logarithms, such an approach is, of course, given full credit.

Question 5: Curve Sketching and Differential Calculus

This question caused the most difficulty to candidates for two reasons; its content and perhaps lack of time.

Drawing/sketching graphs is perhaps the area of the course that results in the poorest responses. It is also the area of the course that results in the best. It is therefore the area of the course that good teaching can influence the most.

Candidates should:

- Use the correct scale and window. Label the axes.
- Enter the formula into the GDC and use the table function to determine the points to be plotted.
- Refer to the graph on the GDC when drawing the curve.
- Draw a curve rather than line segments; ensure that the curve is smooth.
- Use a pencil rather than a pen so that required changes once further information has been gathered (the turning points, for example) can be made.

In part (b) the answer could have been checked using the table on the GDC.

In part (c) **coordinates** were required.

The responses to part (d) were generally correct.

The “show that” nature of part (e) meant that the final answer had to be stated.

The interpretive nature of part (f) was not understood by the majority.

Parts (i) and (j) had many candidates floundering; there were few good responses to these parts.

Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently especially with graphs of functions and statistics.
- Time management – a mark a minute is the guide – ensure that all questions are attempted.
- Cover the whole syllabus; it will all be examined.
- Test students with “show that” that test understanding.
- Ensure candidates label and scale the axes whenever they draw **or sketch** a graph.
- Ensure candidates start each question on a new page and to show all their working.
- Ensure candidates do not write their answers on the question paper.
- Formula booklet should be part of everyday teaching so that candidates become familiar with it.