MATHEMATICAL STUDIES TZ2

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 30	31 – 42	43 – 55	56 – 69	70 – 81	82 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2011 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level, to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

Range and suitability of work submitted

This session there was a wide range of projects in terms of quality of work and topics chosen. Almost all the tasks chosen were appropriate for a Mathematical Studies SL project. There were very few cases where the topic chosen was not an appropriate one and then this was reflected in the analysis part where no, or very few mathematical processes could be applied at all, resulting in more a theoretical project than a mathematical one. The vast majority of candidates opted for a statistical analysis in an attempt to verify a stated hypothesis. Once again, the two main mathematical processes used were Pearson's product moment correlation coefficient and the chi-squared test.

Many students included questionnaires and raw data, but a large number did not, or they organized and presented their data in ways which precluded cross-referencing of data and checking of mathematical processes.

Many projects, where the student collected their own data, did not describe the data collection process in sufficient detail to allow for the assessment of the quality of the data.

A surprising number of students omitted all simple mathematical processes. In this case the first sophisticated process is considered "simple". A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering their test invalid. Students also incorrectly drew conclusions about correlation based on their chi-squared test of independence. Few teachers picked up on these mistakes, suggesting that they are either not checking the accuracy of the math processes in sufficient detail when marking or they also do not understand how to correctly perform a valid chi-squared test or they do not understand the assessment criteria for Criterion C well enough.

Many candidates are now using technology to do the mathematics for them and often do not do any mathematics themselves. Any mathematical processes using technology only is considered simple.

Some teachers appear to be awarding high marks in Criterion F for well-written and organized projects without correct mathematical notation and terminology, suggesting that they clearly do not understand the criterion well enough.

The discussion on validity is still limited mostly to the data collected and many students are not able to demonstrate any understanding of this concept in their projects.

More and more candidates are producing very short projects which do not reflect the 20 hours allocated for school work plus approximately the same amount of time outside of the classroom.

The range of mathematics that was once seen is now significantly diminished.

However, there were some candidates who produced wonderful projects that achieved high levels in almost every assessment criterion.

The comments made by the teachers on the 5/PJCS forms were very clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against the criteria

A. The statement of task was usually evident and most candidates described a plan that they would follow although there was a wide range of detail in the plans submitted. It is important to follow the stated plan. If the plan is well documented, then the project is usually better developed and follows a logical structure. Not all plans were well focused. Some projects did not have a title and, as a result, could not be awarded more than 1 mark for this criterion. Long introductions (including theoretical background of the topic at hand) were often present instead of a plan. Often the "plan" seemed to be more of a summary of the steps taken, written *after* they had in fact been carried out.

B. For the majority of candidates the data was limited in quantity or they did not describe their sample selection and data collection process in sufficient detail to allow for the assessment of the quality of the data. Also, not all candidates set up their data in tables ready for analysis. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted to include this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the mathematical processes used. The quantity of data varied considerably. The candidates must realise that having a lot of data does not always mean that it has the quality needed to gain full marks in this section. If data is too simple then it limits the mathematical analysis that the candidate can perform. When secondary information is used, candidates must clearly identify the source.

C. There were a surprising number of projects which did not include any simple mathematical processes, and for whom the first sophisticated process was counted as a simple one. There were also a large number of projects which included the chi-squared test as the only sophisticated process, but did not use frequency data or did not have sufficient numbers of expected frequencies in each cell of the contingency table to make the test valid. Some candidates only included simple mathematics because their projects did not lend themselves to sophisticated techniques. Many used technology only to perform sophisticated techniques without realizing that this is considered as simple mathematics. Some candidates introduced mathematical processes that were totally irrelevant. This can actually result in the candidate losing marks. Many candidates and their teachers are not clear on the chi-squared test. The entries in the contingency table must be frequencies and the expected frequencies must not be less than 1 and no more than 20% between 1 and 5. Otherwise the test is invalid.

D. Most candidates produced results that were consistent with their analysis. However, few produced detailed discussions. Often this was because the project was too simple and comprehensive discussion was not possible. The stronger candidates did a good job of presenting partial conclusions as they went along and then summarized these to give an overall conclusion at the end.

E. The concept of validity still escapes the weaker candidates. Very few candidates are convincing in their understanding of the notion of validity. Many included the word "validity" in a paragraph, but what they wrote did not demonstrate that they had any understanding of this concept. Their discussions generally centred on data collection. Less often was a student able to comment on the validity of the processes themselves.

F. Most of the projects were well laid out. Many candidates recorded their actions at each stage. It is also important to ensure that the notation and terminology is correct. Many candidates lost marks this session due to errors in either notation or terminology. Some candidates do not seem to be aware that calculator/computer notation is not always correct mathematical notation.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Give them examples of "good" projects so that they know what is expected of them.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the students the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage students to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Emphasise the importance of meeting deadlines
- Inform their students about sampling techniques
- Remind them to include all raw data either in an appendix or as part of the task.
- Show their students how to use Equation editor or Math Type.
- Remind them of the importance of including simple mathematical processes in their projects
- Check the calculations in each project
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 38	39 – 50	51 – 63	64 – 75	76 – 90

The areas of the programme and examination which appeared difficult for candidates

- Truth tables
- Measures of central tendency from frequency tables
- Simultaneous equations
- Concept of a tangent to a curve
- Periodic functions
- Solution of exponential equations
- Producing algebraic equations from given data

The areas of the programme and examination in which candidates appeared well prepared

- Stem and leaf and box and whiskers diagrams
- Converting compound propositions in logic to words
- Coordinate geometry
- Basic probability
- Currency conversion

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Sets

There was much confusion amongst candidates as to the understanding of the words *number of elements*. Many candidates simply wrote down 6 or { 6 } and consequently lost the first mark. Part (b) was done well and many successful attempts were made at completing the Venn diagram in part (c). The most common error in the last part of the question was the omission of the element 10.

Question 2: Statistical measures from a stem and leaf diagram and a box and whisker plot

Whilst this question was done well by many candidates, a significant number miscounted the number of data items required for each measure and, as a consequence, 65, 51.5 and 72 occurred on many scripts. However, virtually all candidates who attempted part (b) secured at least two marks with the available follow-throughs. A small number of scripts showed the last mark lost as the line joining the two endpoints passed through the box.

Question 3: Logic

Many correct answers were seen in part (a) with only a minority of candidates misinterpreting the symbol v as 'and'. Some candidates left out the word 'if' and consequently lost the first mark. Part (b) was not done as well as expected indicating that some work needs to be done by centres on the truth table for the logic symbol $\Rightarrow$. Many correct answers of 'neither' were seen in part (c) but the justification was sometimes lacking definitive reasoning. Without sufficient reasoning, the answer mark was not awarded.

Question 4: Two dimensional mensuration and standard form

A significant number of candidates simply divided 150 000 000 by 365 and consequently lost all but one method mark in part (a). Presumably these candidates assumed that the given value was the circumference rather than the radius. Recovery in part (b) did, however, result in many getting both marks here. It was noted on some answers to part (b) that the index power was negative rather than positive suggesting a misunderstanding by candidates of standard form.

Question 5: Coordinate geometry

Generally, a well answered question with many candidates achieving full marks. Indeed, marks which tended to be lost were as a result of premature rounding rather than method. On a number of scripts, part (a) produced a rather curious wrong answer of 8.2 following a correct gradient expression. It would seem that this was as a result of typing into the calculator 8 – 4 ÷ 5 + 1.

Question 6: Averages and percentage error

In parts (a) and (b), 2.5 was a common incorrect error for both parts as some candidates were confused as to the concept of both the mean and the median from tabular data and simply looked at the mean and median of the *Number of goals*, ignoring the weighting of the number of matches. Candidates faired a little better with part (c) and many correct answers (many as follow through answers) were seen in this part of the question.

Question 7: Simultaneous equations

The first three marks were obtained by a significant majority of candidates. The second equation in x and y proved to be a little more elusive and a popular, but incorrect, answer seen was $12x + 5y = 10000$. Where working was seen in part (d), much of it was wrong. Indeed, a popular, but erroneous method, was to make either x (or y) the subject using one equation and then back substituting the value found into the same equation. Answers, involving decimals, should have flagged to the candidate that something was going wrong somewhere and another look at the question was required. Algebra is always a discriminator on these papers and centres would be well advised to reinforce concepts in such topics.

Question 8: Three dimensional mensuration

As well as some candidates reading the diameter given as the radius, there was much confusion between the area and volume of a sphere and, although there was some recovery when multiplying by 75, two of the three marks were invariably lost. Recovery was possible in part (b) and many successful attempts were seen to calculate the height of the cone.

Question 9: Probability

A reasonably well attempted question with parts (a) and (c) proving to provide many correct answers. A correct answer for part (b) however proved to be a little more elusive as, despite a correct numerator of 25 seen on many scripts, the total sample space was not reduced and a denominator of 250 lost the final mark in this part of the question. On a minority of scripts candidates simply wrote down decimal answers. Where these were correct, both marks for each part were earned. However, incorrect answers earned no marks – candidates would be well advised to at least write down the fraction answer first so that any part marks can be awarded. A case in question here was a predominance of incorrect answers of 0.10 or 10% for part (b). This, on its own earns no marks whereas 25/250 earned A1, A0.

Question 10: Right-angled trigonometry and cosine rule

Although many candidates were able to calculate the size of angle ABD correctly, a significant number then simply stopped, failing to add on 100° and consequently losing the last two marks in part (a). Recovery was seen on many scripts in part (b) as candidates seemed to be well drilled in the use of the cosine rule and much correct working was seen. Indeed, despite many incorrect final answers of 26.4° seen in part (a), many used the correct angle of 126° in part (b).

Question 11: Calculus and tangent drawing

Parts (a) and (b) were reasonably well attempted indicating that candidates are well drilled in the process of differentiation. Correct answers however in part (c) proved elusive to many as frequent attempts to equate the two given functions rather than the gradients of the given functions resulted in a popular, but incorrect, answer of $x = -1.46$. Part (d) was poorly attempted with many candidates simply either not attempting to draw a tangent or drawing it in the wrong place.

Question 12: Percentages and finance

This question was generally well answered with much correct working seen in parts (a) and (b). The most popular incorrect answer in part (a) was 1920 – candidates simply stating the number of defective items rather than the number of non-defective items. Unfortunately in part

(c) many candidates multiplied by 0.8739 rather than divided and 10.49 proved a popular, but erroneous, answer.

Question 13: Periodic sine function

Despite the occurrence of this topic on many previous papers, this question proved to be difficult for many candidates and much incorrect working was seen in parts (a) and (b). Many candidates were able to recover in part (c) gaining at least one mark for correctly placing P on the graph however not as many were able to give an **estimate** of the *x*-coordinate of P. Many answers of 90 were seen and a significant number of candidates gave a solution from their GDC. An estimate was required so any answer of this nature lost this mark.

Question 14: Deriving results from an exponential model

A substituted value of $t = 1$ in part (a) saw many incorrect answers of 23052.70 for this part of the question. Part (b) was better attempted with many correct answers seen. Many candidates picked up the first two marks of part (c) equating a correct expression to half their answer found in part (a). Many though did not seem to know the correct process of using their GDC to find the required answer. Much *trial and improvement* was seen here with varying degrees of success.

Question 15: Constructing a quadratic equation

This question proved to be difficult for the majority of candidates. Many simply were unable to see that, to relate the three given lengths, a Pythagorean equation needed to be produced. Indeed, many did not seem to appreciate the concept of a quadratic equation and, as a consequence, either wrote down a linear equation linking one length to the sum of the other two lengths or multiplied all three lengths together. For the minority who stated a correct Pythagorean equation, many could not remove brackets successfully and arrived at $x^2 = 15$. Consequently, very few candidates earned more than one mark for part (a). Where the correct quadratic equation was seen in part (a), many were able to solve this quadratic correctly in part (b) and arrive at the required value of $x = 5$ for the answer for part (c).

Recommendations and guidance for the teaching of future candidates

Candidates should be encouraged to:

- Give all answers to the required degree of accuracy – although only one accuracy penalty mark and one financial penalty mark was applied to this paper, many candidates lost two marks out of their total because of inaccuracies
- Critically examine their answers to see whether or not they are sensible in the context of the problem set
- Show all working to enable method marks to be obtained if answers are incorrect
- Not cross out their work unless it is to be replaced – crossed out working earns no marks at all

- Practice algebraic manipulation techniques – specifically the expansion of bracketed terms and using factorization to solve quadratic equations
- Ensure that they are fully conversant with the formulae which appear in the information booklet and where exactly these formulae are to be found in the booklet prior to the examination.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 38	39 – 49	50 – 61	62 – 72	73 – 90

General Comments

The paper appeared to be accessible, and of appropriate length. The comments on the G2 forms were mostly appreciative of the syllabus coverage, and of the level of difficulty. Most of the comments indicated that the paper was straight-forward, and that the language was clear. The majority of the candidates demonstrated good knowledge of the course material and ability to apply that knowledge to answer the exam questions.

The areas of the programme and examination which appeared difficult for candidates

The following tasks proved to be challenging for the candidates: Identifying the correct axes of the scatter diagram; using the GDC to find the equation of a regression line; using the GDC to solve quadratic and exponential equations; using the χ^2 value to make a decision about a stated hypothesis in a chi-squared test, and drawing a conclusion about the appropriateness of using a regression line to make an estimation. Candidates had the most difficulty with the trigonometry and the calculus questions. In the former, many candidates incorrectly assumed that the triangle ABC was isosceles or that BN was an angle bisector. In the latter, many candidates had difficulties with identifying the vertical asymptote, using the GDC to identify the coordinates of the minimum and the maximum points, and identifying the range of the function. A great majority of the candidates had difficulty with drawing conclusions and writing clear, succinct, and well grounded justifications to support them. Some candidates lost all marks when they gave incorrect answers without showing their method, and lost the opportunity to gain the method marks. Many students lost a mark due to the accuracy penalty (AP) but it was good to see that only a few candidates were penalized with a unit penalty (UP) for not giving the correct units. There was a noticeable improvement among the candidates in using correct units.

The areas of the programme and examination in which candidates appeared well prepared

The majority of the candidates showed good time management skills and very few scripts had entire questions that were left unattempted. Good working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect. Many scripts were neatly presented although still not all candidates are organizing their working carefully. Most candidates wrote the letter part of the questions next to the working, which is always essential especially in the case of paper 2.

Chi-square test, finding the equation of a regression line, use of a regression line were well understood as were arithmetic and geometric sequences, the mean of a set of numbers, simple and compound interest, currency conversions, and use of the Pythagorean theorem. The degrees of freedom for the chi-squared test were found correctly by most candidates and the null hypothesis was mostly well stated. The sketches of the function in Question 5, and the points plotted on the scatter diagram in Question 1 were mostly satisfactory. Many candidates used the sine rule correctly, and found the area of the triangle in Question 4, and used the derivative of the given function to find the gradient of the graph at a given point in Question 5.

Most candidates were able to demonstrate good knowledge of the learned mathematical concepts and their applications.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Chi-square test and Regression line

Part A: Chi-square test

This question part was answered well by most candidates. The null hypothesis and degrees of freedom were mostly correct. Some candidates offered a conclusion supported by good justifications, but others still showed lack of the necessary knowledge to do that. Some responses to part d) incurred an accuracy penalty for not adhering to the required accuracy level.

Part B: Scatter plot and Regression line

Many candidates reversed the axes in a), but the points were mostly plotted well. The values of the coefficients of the equation of the regression line $y = ax + b$ were often given not to the required 3 significant figure accuracy, and incurred a penalty. The regression line was often drawn not passing through point M and the y-intercept. The responses to the last part of the question were particularly weak, and many candidates were not able to offer a satisfactory reason to support their conclusion.

Question 2: Finance

There was a mix of good and weak responses in part a). Many candidates did not use the correct interest in calculating the interest rate and as a result lost 2 marks. Part b) was well done. Only a handful of candidates were penalized with FP for not giving the answer to the required accuracy level. Parts c) and d) were not answered well. Marks were gained by candidates who showed detailed working. Many candidates had difficulty working with the compound interest formula where the interest was compounded quarterly. Correct final answers in parts c) and d) were rare.

Question 3: Arithmetic and Geometric sequences/series

Part A: Geometric sequences/series

Parts b) and c) were mostly well answered. The majority of the candidates were not able to offer a satisfactory justification in a) and only scored 1 mark. The responses to part d) were often weak. Those candidates who set up the equation scored two marks but very few of them were able to reach the correct final answer.

Part B: Arithmetic sequences/series

Parts a), and b)(i) were mostly answered correctly. Parts b)(ii)a) and b)(ii)b) were poorly answered. Many candidates did not know how to approach the "show that" question. A few were able to solve the quadratic equation using the GDC. Those who attempted to solve it without the GDC generally failed to find the correct answer.

Question 4: Geometry

Many candidates assumed incorrectly that the triangle ABC is isosceles or/and that CN is an angle bisector, and those assumptions led them to use incorrect methods. Wherever those assumptions were made first, all or most of the marks were lost in that specific part of Question 4. There were provisions in the mark scheme to follow through in subsequent parts. Most candidates at least attempted parts a), b) and c). Some candidates incorrectly used an area formula for a right triangle in c) and lost all marks. Many candidates lost a mark for premature rounding in d). Part e) proved to be especially difficult for the candidates. Here many candidates offered guesses instead of sound mathematical reasoning.

Question 5: Calculus

Part a) was either answered well or poorly. Most candidates found the first term of the derivative in part b) correctly, but the rest of the terms were incorrect. The gradient in c) was for the most part correctly calculated, although some candidates substituted incorrectly in $f(x)$ instead of in $f'(x)$. Part d) had mixed responses. Lack of labels of the axes, appropriate scale, window, incorrect maximum and minimum and incorrect asymptotic behaviour were the main problems with the sketches in e). Part f) was also either answered correctly or entirely incorrectly. Some candidates used the trace function on the GDC instead of the min and max functions, and thus acquired coordinates with unacceptable accuracy. Some were unclear that a point of local maximum may be positioned on the coordinate system "below" the point of local minimum, and exchanged the pairs of coordinates of those points in f(i) and f(ii). Very few candidates were able to identify the range of the function in (g) irrespective of whether or not they had the sketches drawn correctly.

Recommendations and guidance for the teaching of future candidates

- *Show working:* All relevant working should be shown in each question. Follow through marks can then be awarded where appropriate
- *When showing work, label the part of the question you are answering:* Proper labeling is necessary as much to help your quick review at the end of the exam as for the examiners when they review and mark your work
- *Use GDC more effectively:* Understand all the relevant functions and uses of the GDC. There is no need to explain how the GDC was used, i.e. which keys were pressed, etc. Candidates need to be encouraged to use their GDC throughout the entire course. Familiarity in using the calculators to graph unfamiliar functions and using it to solve equations is essential
- *Check answers carefully:* Candidates should be reminded to check their answers to ensure they are reasonable in the context of the question
- *Pay attention to the required accuracy for the specific answers:* Candidates should be reminded to give their answers to the accuracy required by a question, or to 3 significant figures otherwise. They must also know what penalties maybe applied if the accuracy is not achieved or the specified units not used
- *Know the command terms:* Students should know all the command terms so that they know what action is required. They should also know the difference between “sketching a graph” and “drawing a graph,” and invest the appropriate efforts in the given task
- *Learn to write succinct, clear, and well grounded justifications:* It is important that students learn to communicate clearly. Teachers should model for students drawing conclusions and writing clear, succinct, and well grounded justifications to support them
- *Review past papers:* Candidates should familiarize themselves with previous papers, their format, and key terms that are used.