MATHEMATICAL STUDIES TZ1

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 16	17 – 31	32 – 45	46 – 58	59 – 71	72 – 83	84 – 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2011 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level, to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 4	5 – 6	7 – 8	9 – 11	12 – 14	15 – 16	17 – 20

Range and suitability of work submitted

This session there was a wide range of projects in terms of quality of work and topics chosen. Almost all the tasks chosen were appropriate for a Mathematical Studies SL project. There were very few cases where the topic chosen was not an appropriate one and then this was reflected in the analysis part where no, or very few mathematical processes could be applied at all, resulting in more a theoretical project than a mathematical one. The vast majority of candidates opted for a statistical analysis in an attempt to verify a stated hypothesis. Once again, the two main mathematical processes used were Pearson's product moment correlation coefficient and the chi-squared test.

Many students included questionnaires and raw data, but a large number did not, or they organized and presented their data in ways which precluded cross-referencing of data and checking of mathematical processes.

Many projects, where the student collected their own data, did not describe the data collection process in sufficient detail to allow for the assessment of the quality of the data.

A surprising number of students omitted all simple mathematical processes. In this case the first sophisticated process is considered "simple". A large number conducted chi-squared tests with insufficient data or non-frequency data, rendering their test invalid. Students also incorrectly drew conclusions about correlation based on their chi-squared test of independence. Few teachers picked up on these mistakes, suggesting that they are either not checking the accuracy of the math processes in sufficient detail when marking or they also do not understand how to correctly perform a valid chi-squared test or they do not understand the assessment criteria for Criterion C well enough.

Many candidates are now using technology to do the mathematics for them and often do not do any mathematics themselves. Any mathematical processes using technology only is considered simple.

Some teachers appear to be awarding high marks in Criterion F for well-written and organized projects without correct mathematical notation and terminology, suggesting that they clearly do not understand the criterion well enough.

The discussion on validity is still limited mostly to the data collected and many students are not able to demonstrate any understanding of this concept in their projects.

More and more candidates are producing very short projects which do not reflect the 20 hours allocated for school work plus approximately the same amount of time outside of the classroom.

The range of mathematics that was once seen is now significantly diminished.

However, there were some candidates who produced wonderful projects that achieved high levels in almost every assessment criterion.

The comments made by the teachers on the 5/PJCS forms were very clear and helpful. Teachers are also encouraged to write on the projects and indicate where the mathematics has been checked for accuracy.

Candidate performance against the criteria

A. The statement of task was usually evident and most candidates described a plan that they would follow although there was a wide range of detail in the plans submitted. It is important to follow the stated plan. If the plan is well documented, then the project is usually better developed and follows a logical structure. Not all plans were well focused. Some projects did not have a title and, as a result, could not be awarded more than 1 mark for this criterion. Long introductions (including theoretical background of the topic at hand) were often present instead of a plan. Often the "plan" seemed to be more of a summary of the steps taken, written *after* they had in fact been carried out.

B. For the majority of candidates the data was limited in quantity or they did not describe their sample selection and data collection process in sufficient detail to allow for the assessment of the quality of the data. Also, not all candidates set up their data in tables ready for analysis. Some candidates had obviously collected data (via a questionnaire or otherwise) but omitted to include this data in their project. If the raw data is not present then the moderator cannot check the accuracy of the mathematical processes used. The quantity of data varied considerably. The candidates must realise that having a lot of data does not always mean that it has the quality needed to gain full marks in this section. If data is too simple then it limits the mathematical analysis that the candidate can perform. When secondary information is used, candidates must clearly identify the source.

C. There were a surprising number of projects which did not include any simple mathematical processes, and for whom the first sophisticated process was counted as a simple one. There were also a large number of projects which included the chi-squared test as the only sophisticated process, but did not use frequency data or did not have sufficient numbers of expected frequencies in each cell of the contingency table to make the test valid. Some candidates only included simple mathematics because their projects did not lend themselves to sophisticated techniques. Many used technology only to perform sophisticated techniques without realizing that this is considered as simple mathematics. Some candidates introduced mathematical processes that were totally irrelevant. This can actually result in the candidate losing marks. Many candidates and their teachers are not clear on the chi-squared test. The entries in the contingency table must be frequencies and the expected frequencies must not be less than 1 and no more than 20% between 1 and 5. Otherwise the test is invalid.

D. Most candidates produced results that were consistent with their analysis. However, few produced detailed discussions. Often this was because the project was too simple and comprehensive discussion was not possible. The stronger candidates did a good job of presenting partial conclusions as they went along and then summarized these to give an overall conclusion at the end.

E. The concept of validity still escapes the weaker candidates. Very few candidates are convincing in their understanding of the notion of validity. Many included the word "validity" in a paragraph, but what they wrote did not demonstrate that they had any understanding of this concept. Their discussions generally centered on data collection. Less often was a student able to comment on the validity of the processes themselves.

F. Most of the projects were well laid out. Many candidates recorded their actions at each stage. It is also important to ensure that the notation and terminology is correct. Many candidates lost marks this session due to errors in either notation or terminology. Some candidates do not seem to be aware that calculator/computer notation is not always correct mathematical notation.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Give them examples of "good" projects so that they know what is expected of them.
- Make sure that they are aware of (and understand) the assessment criteria.
- Remind their students that the project is a major piece of work and should demonstrate a commitment of time and effort.
- Encourage them to think up their own task and explain the plan thoroughly as this gives focus to the task.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Teach the students the significance and limitations of statistical techniques.
- Remind candidates to use only frequencies if they are using the chi-squared test for analysis and check that expected values are more than 5.
- If candidates are using technology then remind them that they are expected to give an example by hand of what they are doing before they start to do any mathematics on the calculator.
- Encourage students to pay more attention to detail such as labels and scales on graphs, spelling mistakes, typos, computer notation.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Emphasise the importance of meeting deadlines
- Inform their students about sampling techniques
- Remind them to include all raw data either in an appendix or as part of the task.
- Show their students how to use Equation editor or Math Type.
- Remind them of the importance of including simple mathematical processes in their projects
- Check the calculations in each project
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Write a comment to justify each achievement level awarded

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 29	30 – 41	42 – 53	54 – 65	66 – 77	78 – 90

General Comments

The paper appeared to be accessible to most candidates. Very few candidates omitted questions or were unable to complete the paper and most showed their working out which enabled follow through and method marks to be awarded, even if they had incorrect earlier responses.

The comments on the G2 forms were mostly appreciative of the syllabus coverage, and of the level of difficulty of the exam paper.

The areas of the programme and examination which appeared difficult for candidates

Numerical calculations (especially multi-step calculations), rearranging formulae, finding angles in 3 dimensional trigonometry, using one variable statistics with a frequency list, completing logic tables, calculating probability for combined events, correcting to 3 significant figures and to the required level of accuracy (as specified in the question), answering simple questions in context (eg the orbit of the satellite), finding the sum of an arithmetic progression, finding the slope of a line perpendicular to a given line and finding the period of a trigonometric function.

In many situations, candidates commenced a question correctly but were unable to achieve the correct answer. This indicated a problem with calculation skills, including the use of their GDC.

The areas of the programme and examination in which candidates appeared well prepared

Use of the correct order of operations with their GDCs, write a number in standard form, recognize an arithmetic progression and find the nth term, complete a probability tree diagram and find simple probabilities, apply Pythagoras theorem, write a linear equation given the slope and a point.

Most candidates were able to demonstrate good knowledge of the content, skills and their application. Working was usually shown and candidates appeared to be aware of the need to provide units, where appropriate.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Substitution of values, significant figures and percentage errors

This question was not well answered by the majority of candidates. Candidates encountered difficulty in part b, not being able to express their answer to one significant figure, or used a mixture of one and two significant figures. Follow through marks in parts bii and c were awarded for candidates who showed their working in calculating p and the percentage error.

Question 2: Bivariate statistics

Part b was not well answered by the majority of candidates, indicating that the use of the GDC is not a natural tool for answering this type of question. Many students ignored the frequencies when finding the mean, median, and standard deviation.

Question 3: Probability

Most candidates were able to answer part a correctly, although many did not realize that 0 is an integer. The award of follow through marks was common in parts b, c and d.

Question 4: Logic

The logic question was clearly difficult for many students. Part a was very poorly done with the majority of students not recognising that two conjunctions were required. Although candidates performed better on part b, many omitted the 'if, (then)'. One of the most common errors in part b was to translate the disjunction as 'and' rather than 'or '.

Question 5: Circumference and standard form

Candidates appeared to be confused by the context in this question. They had difficulty identifying the radius and many used the formula for the area of a circle, rather than the circumference. A large number of candidates misread the final sentence in part b and did not write their answer to the nearest kilometre.

Question 6: Quadratic equations

In part b, the point C was sometimes not labelled or not shown on the graph provided. Candidates using their GDC to find the coordinates of the vertex needed to translate their calculator answer to the exact mathematical answer. Answers of (1.9, 7) or (2.1, 7) did not achieve the maximum number of marks. A common response in part c was to give (4, 3), with no working shown. This incurred a penalty of one mark. The correct answer to this question was $x = 4$

Question 7: Trigonometry

Question 7 was surprisingly difficult for many candidates, especially part b. Many candidates did not recognize that **ACG** was a right angled triangle and tried to use the law of cosines to find angle A. Although correct substitution and manipulation provided the correct answer, many candidates attempting this method made arithmetical errors.

Question 8: Arithmetic sequences and series

Although part a was very well done, a large number of candidates multiplied the difference by n rather than n- 1. Many candidates misread, or misinterpreted, part b and found the 10^{th} term rather than the sum of the first 10 terms.

Question 9: Probability tree diagrams

Part a of this question was well answered, however part b caused many problems. Candidates did not seem to know how to find the probability of the combined events.

Question 10: Linear equations and area of a triangle

In this question, many candidates did not use the x and y intercepts to find the slope and attempted to read ordered pairs from the graph. Part c proved difficult for many candidates, often using trigonometry rather than the more straight forward area of the triangle.

Question 11: Geometric sequences and series

In part a, many candidates gave the common ratio as 3. While they could set up the equation for part c, relatively few succeeded in solving it. Those who arrived at an answer did not always realize that the answer must be an integer.

Question 12: Trigonometric graphs

This question was poorly attempted. Candidates could not find the amplitude or the vertical translation. Many candidates could not find the frequency and appeared to guess the answer for part b. Candidates appeared confused about what part d was asking them to find. Rather than give the interval, many gave the amount of time. Often candidates who had the correct end points did not use the correct interval notation.

Question 13: Linear equations

Parts a and bi of Question 13 appeared to be accessible to most candidates, but part bii was not well attempted. Many candidates did not show their working and lost method marks due to their incorrect answers.

Question 14: Statistics; Box and whisker plots

Question 14 parts a and b were well done. Parts c and d were omitted or incorrectly answered more frequently than any other question on the exam paper.

Question 15: Exponential functions

Very few candidates showed working and subsequently lost marks due to this. Many candidates seemed to forget that $a^0 = 1$ and not 0.

Recommendations and guidance for the teaching of future candidates

Advise candidates to:

- show all relevant working and indicate which part of the question they are answering. Follow-through marks can then be awarded where appropriate
- become familiar with the function and application keys of their calculators, especially for graphing and solving equations
- learn the logic terms such as conjunction, disjunction, if…then
- practice rounding
- check the degree of accuracy required in each question. Answers may be given exactly, to three significant figures or to the accuracy specified in the question
- include units where appropriate
- give answers in financial mathematics questions to the specified accuracy
- learn the command terms to understand the action that is required
- practise writing clear reasons to justify their answers
- gain familiarity with the information booklet

Further comments

Areas of the course which require further emphasis include logic notation, two stage probability, characteristics of trigonometric functions (amplitude, period, frequency, and vertical translations), and finding statistical data using frequency distribution tables. Use of the GDC across the whole curriculum should be encouraged.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 28	29 – 42	43 – 53	54 – 63	64 – 74	75 – 90

General Comments

The majority of candidates attempted all the questions, though there were a number of trivial attempts at questions 3, 4 and 5. It may be that time was an issue for a small minority of candidates as there were scripts wherein only 4 questions showed any working. However, the better candidates were able to display their knowledge and skills over the entire paper, thereby achieving high marks. The examination was deemed to be an appropriate test of the syllabus by the majority of teachers submitting G2 forms, though there were a number of concerns raised about the clarity of wording in question 4; these concerns have been noted.

It was also commented upon that the syllabus coverage was a little uneven; however, syllabus coverage should be judged on both papers as a single examination.

As is ever the case, a number of candidates lost marks in the "show that" questions. It is again reiterated that when candidates are required to reach a given answer that is written to a specified accuracy, they must first write down the value they obtain correct to a higher degree of accuracy and then write down the given value so that these can be seen to be the same.

In the questions asking for angles it is becoming far less common to find candidates using radians; however, the loss of the correlation coefficient due to GDC reset was more of a problem in this paper than has been seen previously.

The areas of the programme and examination which appeared difficult for candidates

- Curve sketching to any degree of accuracy.
- Formal differential calculus.
- Drawing a graph to the correct scale.
- Interpreting and stating the correct conclusion from the results obtained.
- Finance: the difference between simple and compound interest, the difference between interest and the value of an investment, understanding non-annual compounding periods.

The areas of the programme and examination in which candidates appeared well prepared

- Venn diagrams and results obtained from these.

- Chi-squared test on the GDC.
- Probability as read from a contingency table.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Scatter diagram

The great majority of candidates found this question to be a good start to the paper. The common errors were (1) incorrect scales being used; SI units are standard in this course and candidates are expected to know the difference between centimetres and millimetres (2) the lack of r on the GDC (3) not knowing that the regression line y on x passes through the mean point and (4) not realising that the value of r determines the validity of using the regression line y on x.

Question 2: Sets and Contingency Table

Part A

This part was successfully attempted by the great majority. A common mistake was the failure to intersect all three sets.

A surprising number seemed unfamiliar with set notation in (e) and thus did not attempt this part.

Part B

The work on probability also proved accessible to the great majority with a large number of candidates attaining full marks. Most errors occurred due to candidates trying to use the algebraic form of laws of probability, rather than by interpreting the contingency table.

The chi-squared test was well done by the great majority, however, it was clear that a number of centres do not teach this subject, since there were a number of scripts which either were left blank or showed no understanding in the responses seen.

Question 3: Curve sketching and Differential Calculus

As usual and by intention, this question caused the most difficulty in terms of its content; however, for those with a sound grasp of the topic, there were many very successful attempts. Much of the question could have been answered successfully by using the GDC, however, it was also clear that a number of candidates did not connect the question they were attempting with the curve that they had either sketched or were viewing on their GDC. Where there was no alternative to using the calculus, many candidates struggled.

The majority of sketches were drawn sloppily and with little attention to detail. Teachers must impress on their students that a mathematical sketch is designed to illustrate the main points of a curve – the smooth nature by which it changes, any symmetries (reflectional or rotational), positions of turning points, intercepts with axes and the behaviour of a curve as it approaches an asymptote. There must also be some indication of the dimensions used for the “window”.

Differentiation of terms with negative indices remains a testing process for the majority; it will continue to be tested.

It was also evident that some centres do not teach the differential calculus.

Question 4: Finance

The lack of understanding of simple interest by many candidates was somewhat disconcerting; clearly, there is confusion between “interest” paid on and the “value” of an investment for many. There is also confusion about substituting the percentage value into the given simple interest formula.

The evident lack of appreciation of exchange rates and commission is also disturbing; this is clearly not addressed by some centres.

The answer given was intended to be used in the final part of the question, when it is not there is **no** follow through.

Compound interest seems to be better understood by more candidates, however, compounding periods still causes great confusion.

Question 5: Trigonometry, Area and Finance

Most candidates were able to recognise cosine rule, and substitute correctly. Where the final answer was not attained, this was mainly due to further unnecessary manipulation; the GDC should be used efficiently in such a case. Some students used the answer given and sine rule – this gained no credit.

Again, most candidates used the appropriate area formula – however, some did not appreciate the purpose of the given answer and were unable to complete the question accurately.

The final part, in which compound interest was again asked for, tested most candidates but there were many successful attempts using either the GDC’s finance package or correct use of the formula. Care must be taken with the former to show some indication of the values to be used in the context of the question. With the latter approach marks were again lost due to a lack of appreciation of the difference between interest and value.

Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently especially with graphs of functions, statistics and the finance package.
- Teach SI units – if only cm and mm for graphs.
- Ensure that time is spent on sketching a curve given its equation and a calculator screen.
- Ensure candidates label and scale the axes whenever they draw **or sketch** a graph.
- Time management – a mark a minute is the guide – and ensures that all questions are attempted.

- Cover the whole syllabus; it will all be examined – if not in Paper 2 then in Paper 1.
- Practice with "show that" questions; candidates must communicate through their mathematics.
- Ensure candidates start each question on a new page and show their working.