MATHEMATICAL STUDIES TZ2

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 28	29 - 42	43 - 55	56 - 69	70 - 81	82 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2009 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level, to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

Range and suitability of work submitted

Most schools presented quite a wide and interesting variety of topics. Those that are most successful are those where candidates have strong personal involvement. Surveys of schoolmates on random questions do not usually come into this category. Candidates that do achieve personal involvement clearly get enormous pleasure out of their commitment and position themselves to deliver something very enjoyable to read.

However, there were quite a number of schools whose projects all followed not only the same general structure as far as headings, sections and overall layout were concerned, but also applied exactly the same mathematical concepts and techniques to the data collected. This indicates that the teacher may be “blueprinting” the project for their student in a way that precludes the need for any individual creativity or decision making from the candidates.

In some projects the tasks chosen were too narrow but, in most cases, the candidates did have sufficient scope to demonstrate their mathematical ability. As usual statistical projects still predominate. However, it was a pleasure to read some projects based on Mathematical Modelling, Finance, Calculus and Trigonometry.

Most of the projects were well presented with few being hand written. There were a number of very short projects. The internal assessment is intended to be a substantial piece of work and three or four pages of simple mathematics will not score highly in a number of criteria areas. A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the calculations. The quality of the work is very much dependant on the guidance of the teacher.

There was a significant increase in the number of candidates using the chi-squared test and linear regression. A major concern is the number of candidates and teachers who do not realize that not more than 20% of the expected cells can have a number between 1 and 5 and that no expected cells can have a number less than 1 for the test to be valid. Also there is no point in finding a correlation coefficient or regression line if a scatter diagram has shown that there is no correlation. There are still too many projects that look for relationships between variables that don’t make sense from the beginning – foot length v GPA, sleep v alcohol etc – These topics should be discouraged from the start. Students should be steered towards a topic that they are interested in and that has significance.

Another concern is that some candidates are omitting any simple processes. They are going straight into a chi-squared test. They should be aware that this will then be counted as their simple process and they will not be able to score more than 2 in criterion C if this is the only mathematical process that they use in their project. This also means that their discussion of results will be limited. Candidates need to be aware that including simple processes is essential for a good Mathematical Studies project.

More candidates are now using their GDC to do the mathematics for them and often they forget to write down the formula they are using or mention why a particular procedure is used. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

When using the internet the candidate must remember to include the web address in their bibliography.

More teachers are now writing useful and pertinent comments on the cover form and this is useful for the moderation process. However, teachers must ensure that they use the current 5/PJCS form to record the students’ marks.

Candidate performance against the criteria

A. Most of the topics chosen were appropriate for a Mathematical Studies project. The majority of the projects had a title. Many had a clear statement of task and a clear, if not highly detailed, plan. However, there are still some candidates who find it difficult to explain in a clear and concise way their statement of task. When describing the plan, many candidates explain what they are going to do to collect their data, but only some of them describe the mathematical techniques they are going to use in their project. Some candidates did not follow through with their stated plan. In many projects the method used to generate the sample was not specified. The source of the data was unclear in many projects. Candidates with clear statements of task and plan tended to be able to extract more depth from their projects because they knew what they were looking for.

B. The data collected was generally of sufficient quantity but was not always focused on the task. It was easier to find projects where the data could be considered enough in quantity but not in quality. A few candidates did not include raw data within their project or as an appendix, nor did some include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given. It is very difficult for the moderator to check accuracy in cases like this. Also, if a survey or questionnaire is handed out “at random” to a number of people then the candidate should explain what this “at random” means. A large number of candidates simply “dump” tables and charts straight from the internet into their project, with little thought being given to how much of that information is really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way.

C. Most candidates used basic mathematical techniques for analysis, many relying entirely on computer generated results. Many of these candidates omitted explanations and clarifications of these techniques and were not selective about using the particular results that were relevant for their investigations. Computer generated graphs with unlabelled axes were quite common.

   Some candidates are applying sophisticated techniques in their analysis and are omitting the simple mathematics and/or the use of graphs to analyze their information. Hence the range of mathematics applied seemed to be restricted in many projects. With some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. Why have several chi-squared tests in one project? Why find the equation of the regression line when it is clear from the graph that there is no linear relationship? Why find the equation of the regression line and then not use it? Also, the mathematics needs to be done in a meaningful manner. Some projects contained many mathematical calculations, some of which were not relevant for the actual project. The teachers differed in their interpretation of what constituted “sophisticated” mathematical techniques and this was an area that often required moderation.

D. Almost all the candidates were able to produce conclusions or interpretations that were consistent with their analysis but sometimes these were rather brief. In a high number of cases the conclusions were obvious and not very thorough. There is still a tendency to provide subjective reasons for results found that are totally unrelated to any mathematical process carried out.

E. More candidates than in previous sessions commented on validity. Usually this was more to do with the data collection than anything else. A few commented on the mathematical processes that they had used. Of those who did, few reached the level of thoroughness required for a high level of achievement. The stronger candidates are beginning to add sensible suggestions for extensions to the project.

F. Although in a few cases questionnaires used for surveys were sometimes not included in the project and in others it was difficult to follow the process because important data had not been set up for use or had been relegated to an appendix, on the whole, projects were easy to read and well structured. Computer and calculator notation was more widespread than in previous sessions and computer-generated graphs with unlabelled axes were also quite common. Many candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Teachers should ensure that students select subject matter which is both suitable for analysis but also which is of strong personal interest. Proving the obvious is not very motivating for the student.
- They should encourage their students to think through the implications and conclusions from their mathematical processes.
- Teachers need to emphasise the specific purpose of the project.
- The project exercise should be introduced at an early stage in the course to avoid rushed and often poor work handed in just to satisfy a requirement.
- Encourage candidates to use a wide variety of mathematical techniques both simple and sophisticated.
- Advise candidates to collect sufficient data. Often 30 results are not enough for a meaningful analysis.
- Encourage candidates to work on the evaluation area of their project in more depth.
- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.
- Emphasize the importance of showing sample calculations in both simple mathematical processes and sophisticated techniques and to present those calculations, regardless of the use of technology.
- Show comments and corrections on the projects and check the students' calculations.

- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Check that the statement of task does not introduce more than 3 or 4 variables.
- Tell them to state clearly their objectives and to comment on them once the project has been completed.
- Encourage students to pay more attention to detail such as including labels on the axes. Simple mathematical processes such as graphs lack sufficient care in the selection of an appropriate scale and are rendered meaningless by the lack of labelling of the axes or remnants of the default Excel labels.
- Make sure that simple mathematical processes are included
- Try to avoid repeating the same mathematical process several times.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Encourage them to think up their own task and explain the plan thoroughly.
- Advise the candidates to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Monitor students' work closely and give them hints or suggestions which might lead them to more creative applications of their knowledge.
- All sources should be properly documented.

- Collect two copies of the project from each candidate so as to make sure that the moderator gets an original version and not a copy.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 24	25 - 34	35 - 47	48 - 59	60 - 72	73 - 90

General Comments

This paper appeared to be accessible to most candidates with many demonstrating a sound knowledge of the course and an ability to apply their knowledge in a wide range of contexts. Time did not appear to be a problem as most candidates attempted all questions, however many candidates did not attempt all parts of each question.

The majority of candidates lost a unit penalty mark but too many candidates lost two or three penalty marks. Candidates used their graphic display calculator effectively, although a number did not use the GDC to its full capacity and therefore lost time in answering questions manually. Many candidates neglected to round off their calculator answers to three significant figures.

The areas of the programme and examination which appeared difficult for candidates

Statistics questions proved difficult for this group of candidates with many not knowing the difference between discrete and continuous data. A significant number of candidates tried to work out the mean and the equation of the regression line by hand, while question 3, which involved finding the mean, median and mode from a frequency table, created problems for many.

Question 8, with compound and simple interest proved difficult as many candidates were confused between total amount earned, interest earned and how you change the interest rate and time period when the interest is not compounded annually. For those candidates who showed their working, follow through marks could be awarded.

Many candidates had problems finding the gradient of a line perpendicular to a given line. Question 9(b)(ii) proved to be very difficult with many candidates unable to find the value of p.

The calculus question, particularly part (c) created difficulties with many candidates not realising they had to equate their answers to (a) and (b) to find the value of x.

Question 13 with conditional and two stage probability proved difficult for many candidates and a surprising number expressed their probability answers as numbers greater than one.

Few candidates could find the equation of the parabola in question 14 (b), although most could find the correct answer to part (a).

The areas of the programme and examination in which candidates appeared well prepared

The majority of candidates showed their working so that method marks and follow through marks could be awarded, even when parts of the questions were incorrect.

Perimeter and area of a rectangle, substituting values into an equation, arithmetic sequences and series, finding the gradient of a line joining two points, simple probability, differentiation, currency conversion and right-angled trigonometry were all well answered.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Perimeter and Area of a Rectangle, Standard Form

This question was well answered by many candidates although the majority lost a mark as a unit penalty in part (a). Some candidates used the wrong formula for the perimeter. Most could give their answer in standard form.

Question 2: Linear Cost Function

The majority of candidates answered this question correctly. Some gave an answer of 150 AUD for part (a). Very few lost a unit penalty mark in part (c).

Question 3: Statistics

This question was not well answered by a number of candidates. In part (a), many did not know that the data was discrete. The most common error in part (b) was $(0+1+2+3+4)/5=2$.

Many candidates did not know how to find the median and the most common incorrect answer for part (d) was 11.

Question 4: Arithmetic Sequence

Most candidates recognised the arithmetic sequence and used the correct formula, although some used a list to find the answers. A common error was to use the common difference as 2 rather than -2. Many candidates were awarded follow through marks in part (b), correctly using their incorrect value of n from part (a).

Question 5: Cosine graph

Very few candidates could give the period of this function although most knew the amplitude. They then found difficulty relating these answers to the values of a and b. Follow through marks could be awarded from their answer to (a)(i) for (b)(ii). Many candidates omitted the negative sign in part (b)(i).

Question 6: Linear Regression

Some candidates attempted to find the equation by hand, generally without success. Those who used their calculator could quickly find the equation and use it to find the number of ice cream sales. A significant number of candidates lost one mark for writing the equation with y and x rather than s and t. A lesser number lost the accuracy mark for an integral number of ice-creams.

Question 7: Coordinate Geometry

Most candidates could find the x and y intercepts but many wrote the coordinates the wrong way around. A number of candidates did not label their coordinates as P and Q or did not include parentheses. In part (b) many had trouble recognising the need to solve the two simultaneous equations.

Question 8: Compound and Simple Interest

This is a question that has been tested before but few candidates managed to gain full marks. Compounding the interest quarterly and using the correct compound interest formula appeared to challenge many candidates. In part (b) many did not subtract $\$4500$, when using simple interest. Very few candidates lost the financial penalty mark in this question.

Question 9: Gradients of lines

While parts (a) and (b)(i) were attempted with some success, few candidates made progress in (b)(ii). Some candidates used the coordinates of point B rather than C and others could not find the unknown value p as they did not realise they had to equate their substituted formula for the gradient to the answer to part (b)(i). A large number of candidates did not attempt this part of the question.

Question 10: Probability

This question proved to be difficult with many candidates unaware of the significance of mutually exclusive events in probability. A significant number gave the answer to (b) as the answer to (a).

Part (c) proved to be difficult for some but most of the candidates who used the formula were able to achieve full marks. Very few candidates used Venn diagrams to answer this question.

Question 11: Differentiation

This question was generally answered well in parts (a) and (b). Part (c) proved to be difficult as candidates did not realise that to find the value of the x coordinate they needed to equate their answers to the first two parts. They did not understand that the first derivative is the gradient of the function. Some found the value of x, but did not substitute it back into the function to find the value of y .

Question 12: Currency Conversion

This question was well answered by many candidates, particularly part (a), however a significant number lost the financial penalty mark for not giving an answer correct to two decimal places, as stated in the question.

Question 13: Probability

The answers $1/8$ and $3/8$ were provided by many rather than 1 and 3. The conditional probability question was correctly answered more often when the formula was used. A common incorrect answer to part (c) was $3/8\times2/7$.

Question 14: Quadratic Function

Most candidates successfully found h but very few could find the equation of the curve.

Question 14 (b) appeared to be the most difficult question on the paper.

Question 15: Three dimensional Trigonometry

This question was well answered by many candidates although a number lost an accuracy penalty or a unit penalty in this question. A very common error was assuming PB to be 20cm. The mark-scheme allowed for follow through marks to be awarded in this case. Most candidates could find the angle and very few did not use right angled trigonometry.

Recommendations and guidance for the teaching of future candidates

Learn the Command Terms:

Candidates should know all the command terms so that they are aware that when a question says ‘write down’, no calculations are necessary.

Avoid losing Penalty Marks:

Teachers should remind candidates to express answers to the accuracy required in a question, or to 3 significant figures, to avoid incurring an accuracy penalty. Candidates must also be encouraged to include units in all answers to avoid a unit penalty and to express answers to financial questions to the accuracy stated in the question (financial penalty).

Round Answers appropriately:

Premature rounding results in incorrect answers. Teachers should remind candidates not to round their answers too early but to ensure their answers are given to the appropriate accuracy, as specified in the question, or to three significant figures.

Show working:

All relevant working should be shown in each question with the question part indicated in the working box. Follow through marks can then be awarded where appropriate.

Use their GDC more effectively:

Understand all the relevant functions of their GDC, including finding the mean, regression equation and point of intersection of two lines.

When giving an answer from their GDC, candidates should provide evidence of their work.

Cover all topics on the syllabus:

The whole syllabus needs to be taught as exam setters write questions from all areas of the course.

Check answers carefully:

Candidates should be reminded to check their answers to ensure they are reasonable in the context of the question.

Practise Past Papers:

Candidates should have as much practice as possible in answering questions written in different styles.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 43	44 - 53	54 - 64	65 - 74	75 - 90

General Comments

Most candidates attempted all the questions and were given an excellent chance to demonstrate what they had learnt as it was clearly reflected in most of the comments sent in the G2s. It was also clear that time was not an issue for candidates and that the better ones could display their knowledge and skills thus achieving high marks. It was considered a simpler although reasonable paper than previous years with the majority of teachers considering it quite appropriate.

Accuracy and unit penalties were applied in many cases; although not as frequently as in the past. It is essential that teachers stress the importance to candidates of writing their answers exact or correct to three significant figures, and of writing down the units next to their answers.

A good number of candidates lost marks in the “show that” parts of questions 2 and 5. When candidates are required to reach a given answer that is written to a specified accuracy, they must write down that value with a higher degree of accuracy (unrounded value). Also frequently premature rounding resulted in the marks being lost.

In the questions asking for angles it was less common to find candidates using their GDC in radians; however, there are still a good number of them that did not realise that they should have changed the mode back to degrees. Although follow through marks were awarded, in some cases marks were lost because of being either negative or unrealistic.

In question 2 good graphs were generally seen with axes correctly labelled and with proper scales. There were still candidates that have not drawn a smooth curve and also, some that drew such a small graph that it made it impossible to follow through their values.

Despite incorrect answers, follow through marks were given when proper working was shown.

The areas of the programme and examination which appeared difficult for candidates

- Conditional probability (question 1)
- Non-right angle trigonometry (question 3) caused difficulties for many, with triangles assumed to be right-angled or isosceles. Follow through marks were essential here.
- Geometric sequence (question 4)
- Finding the derivative of $\frac{5}{x^2}$ (question 5)
- Showing how to obtain expected values (question 2)
- “Show that” questions (question 5)

The areas of the programme and examination in which candidates appeared well prepared

- Venn diagrams
- Chi-squared test
- Drawing a graph

- Logic
- Use of GDC to find the minimum value and intersections

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Venn Diagram / Probability

(a) Most candidates began the paper well by correctly drawing the Venn diagram and answering parts (b) and (c) correctly.

(d) Conditional probability has proved difficult for many candidates; only a very small part of the candidates scored full marks for this part.

Question 2: Chi-squared test/Exponential function

It was clear that the candidates who performed poorly in part (i) lacked the basic knowledge of chi-squared analysis. Some mixed up the null and alternate hypotheses and also were not able to correctly demonstrate the way of finding the expected value. There were many errors in finding the critical value of χ^2 at the 1% level of significance.

(ii) Candidates found this part rather easy, with some making arithmetic mistakes and thus losing one or more marks. The graph was well done with a high percentage scoring full marks. Some candidates did not label the axes, others had an incorrect scale and a few lost one mark for not drawing a smooth curve.

Question 3: Trigonometry

(a) and (b) This was a simple application of non-right angled trigonometry and most candidates answered it well. Some candidates lost marks in both parts due to the incorrect setting of the calculators. Those that did not score well overall primarily used Pythagoras.

(c) and (d) Most candidates scored full marks, many by follow through from an incorrect part (b). The main error was using the value for BC and not BD.

(e) Done well; again some candidates used the right-angled formula.

(f) This part was poorly done; many candidates unable to convert 3cm to 0.03m. A significant number used the wrong formula, multiplying their answer by $1/3$.

Question 4: Sequences/Logic

(i) (a) An easy ratio to find and the majority of candidates found $r = 3$, though many had trouble showing the appropriate method, thus losing marks.

(b) A fairly straightforward part for most candidates.

(c) The majority found $k-7$; many without supporting work which lost them a mark. Where candidates had difficulty in this part, it was generally a case of poor algebraic skills.

(ii) This question on logic was straightforward for most candidates who scored full marks for parts (a) and (b) (i). A few omitted the brackets in part (b).

(ii) (b) (ii) Very poorly answered with many candidates scoring just one mark. The main error was to open the bracket and not use the “or”.

Question 5: Calculus

A large number of candidates were not able to answer this question well because of the first term of the function’s negative index. Many candidates who did make an attempt lacked understanding of the topic.

(a) Most candidates could score 4 marks.

(b) A good number of candidates correctly substituted into the original function.

(c) Very few managed to answer this part algebraically. Those candidates who were aware that they could read the values from their GDC gained some easy marks.

(c) (ii) Proved to be difficult for many candidates as increasing/decreasing intervals were not well understood by many.

(d) As this part was linked to part (a), candidates who could not find the correct derivative could not show that the gradient of T was -7. Many candidates did not realize that they had to substitute into the first derivative. For those who did, finding the equation of T was a simple task.

(e) For those who had part (d) (ii) correct, this allowed them to score easy marks. For the others, it proved a difficult task because their equation in (d) would not be a tangent.

Recommendations and guidance for the teaching of future candidates

- Ensure candidates can use the GDC efficiently
- Practice past IB questions and timed exam papers
- Time management – a mark a minute
- Cover the whole syllabus
- Teach candidates to read the questions properly, identify topic and highlight main points

- Check that candidates' answers are reasonable
- Practice with "show that" questions
- Familiarize candidates with AP´s, UP´s and FP´s penalties.
- Make sure candidates label and scale the axes each time they draw a graph
- Show candidates the importance of starting each question on a new page and to show all their working.
- Candidates should not write down which GDC keys they used to find an answer; marks will not be awarded for it.
- Formula booklet should be part of every day teaching so that candidates become familiar with it.
- GDC should be used daily so that candidates can become familiar with their use, especially with graphs of functions and statistics.