MATHEMATICAL STUDIES TZ1

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 15	16 - 28	29 - 39	40 - 53	54 - 66	67 - 79	80 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2009 examination session the IB has produced time zone variants of the Mathematical Studies papers. Grade boundaries for the different time zoned papers are set separately, and careful judgments are made that are based on criteria for performance level, to account for differences in the papers.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

Range and suitability of work submitted

Most schools presented quite a wide and interesting variety of topics. Those that are most successful are those where candidates have strong personal involvement. Surveys of schoolmates on random questions do not usually come into this category. Candidates that do achieve personal involvement clearly get enormous pleasure out of their commitment and position themselves to deliver something very enjoyable to read.

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However, there were quite a number of schools whose projects all followed not only the same general structure as far as headings, sections and overall layout were concerned, but also applied exactly the same mathematical concepts and techniques to the data collected. This indicates that the teacher may be "blueprinting" the project for their student in a way that precludes the need for any individual creativity or decision making from the candidates.

In some projects the tasks chosen were too narrow but, in most cases, the candidates did have sufficient scope to demonstrate their mathematical ability. As usual statistical projects still predominate. However, it was a pleasure to read some projects based on Mathematical Modelling, Finance, Calculus and Trigonometry.

Most of the projects were well presented with few being hand written. There were a number of very short projects. The internal assessment is intended to be a substantial piece of work and three or four pages of simple mathematics will not score highly in a number of criteria areas. A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the calculations. The quality of the work is very much dependant on the guidance of the teacher.

There was a significant increase in the number of candidates using the chi-squared test and linear regression. A major concern is the number of candidates and teachers who do not realize that not more than 20% of the expected cells can have a number between 1 and 5 and that no expected cells can have a number less than 1 for the test to be valid. Also there is no point in finding a correlation coefficient or regression line if a scatter diagram has shown that there is no correlation. There are still too many projects that look for relationships between variables that don't make sense from the beginning – foot length v GPA, sleep v alcohol etc – These topics should be discouraged from the start. Students should be steered towards a topic that they are interested in and that has significance.

Another concern is that some candidates are omitting any simple processes. They are going straight into a chi-squared test. They should be aware that this will then be counted as their simple process and they will not be able to score more than 2 in criterion C if this is the only mathematical process that they use in their project. This also means that their discussion of results will be limited. Candidates need to be aware that including simple processes is essential for a good Mathematical Studies project.

More candidates are now using their GDC to do the mathematics for them and often they forget to write down the formula they are using or mention why a particular procedure is used. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

When using the internet the candidate must remember to include the web address in their bibliography.

More teachers are now writing useful and pertinent comments on the cover form and this is useful for the moderation process. However, teachers must ensure that they use the current 5/PJCS form to record the students' marks.

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Candidate performance against the criteria

A. Most of the topics chosen were appropriate for a Mathematical Studies project. The majority of the projects had a title. Many had a clear statement of task and a clear, if not highly detailed, plan. However, there are still some candidates who find it difficult to explain in a clear and concise way their statement of task. When describing the plan, many candidates explain what they are going to do to collect their data, but only some of them describe the mathematical techniques they are going to use in their project. Some candidates did not follow through with their stated plan. In many projects the method used to generate the sample was not specified. The source of the data was unclear in many projects. Candidates with clear statements of task and plan tended to be able to extract more depth from their projects because they knew what they were looking for.

B. The data collected was generally of sufficient quantity but was not always focused on the task. It was easier to find projects where the data could be considered enough in quantity but not in quality. A few candidates did not include raw data within their project or as an appendix, nor did some include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given. It is very difficult for the moderator to check accuracy in cases like this. Also, if a survey or questionnaire is handed out "at random" to a number of people then the candidate should explain what this "at random" means. A large number of candidates simply "dump" tables and charts straight from the internet into their project, with little thought being given to how much of that information is really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way.

C. Most candidates used basic mathematical techniques for analysis, many relying entirely on computer generated results. Many of these candidates omitted explanations and clarifications of these techniques and were not selective about using the particular results that were relevant for their investigations. Computer generated graphs with unlabelled axes were quite common.

D. Some candidates are applying sophisticated techniques in their analysis and are omitting the simple mathematics and/or the use of graphs to analyze their information. Hence the range of mathematics applied seemed to be restricted in many projects. With some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. Why have several chi-squared tests in one project? Why find the equation of the regression line when it is clear from the graph that there is no linear relationship? Why find the equation of the regression line and then not use it? Also, the mathematics needs to be done in a meaningful manner. Some projects contained many mathematical calculations, some of which were not relevant for the actual project. The teachers differed in their interpretation of what constituted "sophisticated" mathematical techniques and this was an area that often required moderation.

E. Almost all the candidates were able to produce conclusions or interpretations that were consistent with their analysis but sometimes these were rather brief. In a high number of cases the conclusions were obvious and not very thorough. There is still a tendency to provide subjective reasons for results found that are totally unrelated to any mathematical process carried out.

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F. More candidates than in previous sessions commented on validity. Usually this was more to do with the data collection than anything else. A few commented on the mathematical processes that they had used. Of those who did, few reached the level of thoroughness required for a high level of achievement. The stronger candidates are beginning to add sensible suggestions for extensions to the project.

G. Although in a few cases questionnaires used for surveys were sometimes not included in the project and in others it was difficult to follow the process because important data had not been set up for use or had been relegated to an appendix, on the whole, projects were easy to read and well structured. Computer and calculator notation was more widespread than in previous sessions and computer-generated graphs with unlabelled axes were also quite common. Many candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

H. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Teachers should ensure that students select subject matter which is both suitable for analysis but also which is of strong personal interest. Proving the obvious is not very motivating for the student.
- They should encourage their students to think through the implications and conclusions from their mathematical processes.
- Teachers need to emphasise the specific purpose of the project.
- The project exercise should be introduced at an early stage in the course to avoid rushed and often poor work handed in just to satisfy a requirement.
- Encourage candidates to use a wide variety of mathematical techniques both simple and sophisticated.
- Advise candidates to collect sufficient data. Often 30 results are not enough for a meaningful analysis.
- Encourage candidates to work on the evaluation area of their project in more depth.
- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.
- Emphasize the importance of showing sample calculations in both simple mathematical processes and sophisticated techniques and to present those calculations, regardless of the use of technology.
- Show comments and corrections on the projects and check the students' calculations.

- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Check that the statement of task does not introduce more than 3 or 4 variables.
- Tell them to state clearly their objectives and to comment on them once the project has been completed.
- Encourage students to pay more attention to detail such as including labels on the axes. Simple mathematical processes such as graphs lack sufficient care in the selection of an appropriate scale and are rendered meaningless by the lack of labelling of the axes or remnants of the default Excel labels.
- Make sure that simple mathematical processes are included
- Try to avoid repeating the same mathematical process several times.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Encourage them to think up their own task and explain the plan thoroughly.
- Advise the candidates to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.
- Monitor students' work closely and give them hints or suggestions which might lead them to more creative applications of their knowledge.
- All sources should be properly documented.

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- Collect two copies of the project from each candidate so as to make sure that the moderator gets an original version and not a copy.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 37	38 - 49	50 - 60	61 - 72	73 - 90

General Comments

This paper proved to be accessible to the majority of candidates. Time did not appear to be an issue for candidates, though the weakest scored poorly on the later questions in the paper. The comments on the G2 forms were largely encouraging, the great majority judging the paper's level to be appropriate, if perhaps a little easier than has latterly been the case. As usual, there was a wide range of marks (from zero up) with the great majority of candidates from some schools scoring well. However, other centres did not prepare their candidates properly for this examination as a number of topics were not attempted by all students. The treatment of accuracy and unit penalties again showed an improvement over previous years but the financial penalty was a pitfall for many; there are still too many candidates that are not following accuracy instructions. The graphic display calculator (GDC) was not always used effectively, most notably in the solution of simultaneous equations. The questions that posed the most problems were questions 6, 7, 14 and 15.

The areas of the programme and examination which appeared difficult for candidates

The GDC was not used properly or to its full capability in answering some of the questions; too many candidates ignored it in question 1, question 5(c) and question 14(b).

The financial and the geometric progression questions caused problems for many, as did the questions that included the use of parameters and some algebraic manipulation. Trigonometric and exponential functions remain an area of weakness. The interpretive nature of the chi-squared test was a challenge for many candidates; lack of understanding of the process of hypothesis testing abounded and this should be addressed. There was confusion between domain and range. Conditional probability remains one of life's mysteries for much of the candidature.

The areas of the programme and examination in which candidates appeared well prepared

Working was shown by the majority of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect.

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The questions on logic, arithmetic sequences (though the series was less well understood) and basic statistics were well answered by the majority of the candidates.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Statistics from raw data

It was expected that candidates would use the GDC for this question.

(iv) It was pleasing to note that most candidates were able to choose the appropriate standard deviation.

(b) A common error was writing the IQR as an interval.

Question 2: Logic

(a) The most common error was poor use of the “If...then” connective.

(b) Confusion between “and” and “or” was rare, however, the use of implication in this part was a little too common.

(c) Precise, correct terminology was expected in this part.

Question 3: Arithmetic sequence and series

(a) This was generally well attempted, often by listing.

(b) The common misconception was the use of the “term by term” formula. Listing the values in this part was usually not a successful strategy. The incorrect substitution of the answer to (a) also led to errors.

Question 4: Gradient of straight line and trigonometry

(a) This was generally well answered.

(b) This, too, was generally well answered, the errors coming from incorrect substitution into the gradient formula rather than using the two intercepts.

(c) There was often a lack of accuracy in the answers. Also, the use of the sine rule overly complicated matters for many.

Question 5: Factorising and functions

(a) This was generally well answered, but a number seemed not to know the term “factorise”.

(b) This, too, was generally well answered.

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(c) This part also proved problematic for many candidates. It was expected that the GDC was used, though many attempted an algebraic solution.

Question 6: Chi-squared test of independence

This question caused significant difficulties for many candidates. It seemed, from the responses, that the purpose of the test is not well understood, even if its procedure on the GDC can be performed.

(a) The test is one of “independence” and it should be stressed to candidates that it is this which is key in stating the hypotheses. Improper terminology, most notably, “not correlated” is not acceptable.

(d) Many candidates did not know the correct figures to compare in order to arrive at the decision. Others gave no reason at all.

Question 7: Quadratic function

(a) The lack of the answer, “ $x=3$ ”, expressed as an equation was a common fault.

(b) Misreading the y coordinate was the common error.

(c) This part proved challenging for many; there was confusion between domain and range for many, the incorrect inequality also was a common error. It is the accepted practice in examinations that if a domain is not specified, then it is taken as the real numbers.

Question 8: Geometric sequence and series

Many marks were lost through incorrect rounding or premature rounding (if a year by year approach was used).

(a) This part was well attempted, errors being the use of 4% as the common ratio.

(b) The common error here was the use of the incorrect index in the formula.

(c) Attempts at calculation without use of the formula were largely unsuccessful.

Question 9: Sine graph

This question was either well attempted (by the majority, in fact) or very poorly. The common errors in (c) were either the listing of the solutions or the lack of appreciation of the domain. It should be noted that radian measure is **not** used in the MSSL course.

Question 10: Sine rule and percentage error

(a) Use of cosine rule was common. The assumption of a right angle in the given diagram was minimal.

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(b) The incorrect denominator was often seen in the error formula.

Question 11: Probability

The diagram caused some difficulty for some candidates, however the majority of candidates were successful in (a).

(b) The term “difference” was well understood by the candidature.

(d) Caused the most difficulty. Candidates are still unable to interpret conditional probability. Many use the formula without understanding. An intuitive approach using diagrams is still encouraged.

Question 12: Simple and compound interest

This question was poorly answered by many of the candidates. Candidates confuse interest with principal in the formulas.

(a) Rates of interest were not entered as percentages.

(b) The idea of compounding periods and the implication for determining the level of interest is poorly understood. The correct answer is 3.5 years. Interpretation of compounding periods is expected.

Question 13: Tennis balls and their container

This question was poorly answered by many but perfectly well by many others, there being little in between. Volume seemed to be little understood and this part of the course is perhaps overlooked. A (candidate drawn) diagram helped visualise the situation and this, in general, is to be encouraged. Many found (b) difficult due to it not being broken up into “one stage” parts in the question. Practice in multi-stage questions is recommended.

Question 14: Exponential Function

Concern was expressed about the wording of the question; answers were given great leeway by examiners and suggestions for wording are welcome. The 24 hour clock method of describing time is the norm in IB examinations. It should be recognised that the purpose of this question was to discriminate at the grade 6/7 level.

(a) The concept of the zero index was not understood by many.

(b) The use of the GDC was (as always) expected in solving the simultaneous equations.

(c) Working was required for follow through in this part.

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Question 15: Differentiation

The structure of this question was not well understood by the majority; the links between parts not being made. Again, this question was included to discriminate at the grade 6/7 level.

(a) Most were successful in this part.

(b) This part was usually well attempted.

(c) Only the best candidates succeeded in this part.

Recommendations and guidance for the teaching of future candidates

The candidates need to gain more confidence in using their GDC. Further practice with both the numeric and graphical functions of the GDC would benefit all candidates. Quadratic and simultaneous equations should be solved on the GDC either with the built in functions or, in this case, graphically.

It is further expected that the great majority of statistical calculations will require the use of the GDC. Candidates should be reminded that errors in inputting data cannot be addressed by markers since no working is shown – hence **great** care must be taken and the entries checked.

The broad areas of the entire syllabus will be tested over both papers (though not necessarily all in each paper) and it should all be taught. It may well be that specific parts of the syllabus domains – regression, in this case – may be omitted.

Teachers need to remind candidates of the three types of penalty; accuracy, financial and that for units being omitted or incorrect.

All relevant working should be shown in each question so that follow through marks can be awarded when necessary. With the more frequent use of the financial application (TVM solver), it should be emphasised that the only acceptable working that can be shown is via the substituted formulas.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 33	34 - 46	47 - 58	59 - 71	72 - 90

General Comments

The paper proved to be fair and straightforward with most candidates managing to attempt all the questions. It was clearly reflected in the G2´s that the level of difficulty of the paper was appropriate and of the similar standard of previous May session´s Paper 2. Teachers and examiners' comments supported the idea that time did not seem to be a factor. Also it was mentioned that questions were nicely written, had a reasonable length and could therefore be completed in the time given by the average Mathematical Studies student.

Accuracy and unit penalties were applied in many cases though not with the same frequency as previously. It is clear that teachers are highlighting in their lessons the importance of writing their answers correct to three significant figures or exact, and writing down the units next to their answers.

Many candidates lost the last answer mark in the "show that" part question of question 4. When candidates are required to reach a given answer that is written to a specified accuracy, they must write down that value with a higher degree of accuracy (unrounded value). Also frequently premature rounding resulted in the final marks being lost.

It was not common to find a candidate with their calculator set in radians. However, there still some that forget to set the mode back into degrees resulting in some answer marks lost for being either negative or unrealistic.

Very good graphs were seen in question 5: scales on axes and axes labelled, correct shape, axes intercepts and minimum in the correct position but it was expected to see more of these. It is clear that some students are still finding it difficult to draw graphs even with the help of the graphics display calculator and also with the required window stated in the question. Many students lost one mark for not labelling the axes and/or not stating the scales on them. In general very tiny graphs do not gain full marks as some of the features of the graph are lost due to its size.

Where clear working was shown, follow-through and method marks could be awarded when the answers were incorrect.

Many centres sent the exam booklet with the student´s answers on them and also in the answer sheets. This makes the marking difficult and also may lead to a situation in which the student could be disadvantaged with some marks lost due to the lack of tidiness and organization of their answers.

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The areas of the programme and examination which appeared difficult for candidates

- Using midpoint values to find mean and standard deviation of grouped data
- Providing complete explanations to “show” a given result
- Conditional probability
- Volume of a prism
- Notation and meaning of derivative function
- Connection between calculus and graph of a function
- Tangent (and horizontal) to a graph

The areas of the programme and examination in which candidates appeared well prepared

- Interpreting a cumulative frequency graph
- Interpreting Venn diagram
- Area of triangle
- Cosine rule
- Tree diagram - Simple probability calculations
- Coordinate geometry

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Cumulative frequency graphs - Mean and standard deviation for grouped data

Most of the students knew the definition of the median, quartiles and inter-quartile range though some confused variables and worked with the frequencies instead. Few could use their calculator to estimate the mean and standard deviation from grouped data. It cannot be said that the calculator was misused but that frequencies and midpoints were ignored when doing the calculations. Part (f) acted as a good discriminator. Follow through marks were awarded in (f) when working was shown.

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Question 2

(i) Probability-Tree diagram

The tree diagram was quite well answered by many students, but sometimes it was missing on many papers. It seemed they had it on their examination paper because the subsequent questions were answered accurately. Conditional probability was of great difficulty to many candidates.

(i) Sets-Venn diagrams

This question was well handled although part (d) proved too difficult for many candidates and demonstrated, overall, a poor level of understanding of basic set notation. Students generally had the algebraic skills required to solve for x in part (e)(ii).

Question 3: Coordinate geometry

This was well done overall. Almost all students could calculate the gradient of the straight line. Gradient of perpendicular line was found, but some candidates failed to communicate the requirement, in terms of gradients for two lines to be perpendicular (Example: They are perpendicular because their gradients are opposite and reciprocal or they are perpendicular because the product of their gradients is -1)

Distance between points and area of triangle was answered well by most candidates. Both formulae for the area of the triangle were correctly used.

Question 4: Trigonometry

It was pleasing to show candidate working throughout this question. Follow through marks could be awarded when incorrect answers were given. Many candidates incorrectly calculated the weight of the chocolate bar by multiplying the surface area by $1.5g$. Also a large number of students incorrectly used the formula for the volume of a pyramid rather than for a prism. Most candidates were successful in their use of the cosine rule but did not give the answer before it was rounded to 86.4, resulting in the loss of the final A mark. The last part acted as a clear discriminator, very few students were able to find the correct length of the new chocolate bar. Most students used units correctly.

Question 5: Calculus

Many students did not know the term “differentiate” and did not answer part (a). However, the derivative was seen in (b) when finding the gradient at $x=1$. The negative index of the formula did cause problems for many when finding the derivative. The meaning of the derivative was not clear for a number of students. Part (d) was handled well by some but many substituted $x=0$ into $f'(x)$. It was clear that most candidates neither knew that the tangent at a minimum is horizontal nor that its gradient is zero. There were good answers to the sketch though setting out axes and a scale seemed not to have had enough practise.

Those who were able to sketch the function were often able to correctly place and label the tangent and also to find the second intersection point with the graph of the function.

Recommendations and guidance for the teaching of future candidates

- Teach better understanding of what is required to ´show that`.
- Applying AP´s, UP´s and FP´s should be part of a teacher routine in marking throughout the 2 years course.
- Remind students to label and scale the axes each time they draw a graph
- Students need to monitor their time more carefully and need more guidance on how to pace themselves during this 90 minute exam (1 mark per minute).
- Tell your students to start each question on a new page and to show all their working.
- Tell your students not to write down which GDC keys they used to find an answer as this does not gain any marks and is a waste of precious time.
- The information booklet needs to be used throughout the 2 years course on a daily basis.
- Give your students the opportunity to practice more with the GDC. More emphases need to be put on graphs of functions and statistics.
- Remind your students to write their answers in the answers sheet provided and **<u>not</u>** in the exam paper.

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