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MATHEMATICAL STUDIES TZ2

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 24	25 - 35	36 - 49	50 - 64	65 - 77	78 - 100

Time zone variants of examination papers

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2008 examination session the IB has produced time zone variants of the Mathematical Studies papers.

Introduction

Mathematical Studies SL continues to increase its candidature with over 17000 students taking the examination this May.

This was the first year that Mathematical Studies SL has been regionalized and the grade award meeting contained senior examiners who had marked at least two different components. This meant that for each component of the external papers the grade boundaries were carefully found using the grade descriptors with examiners who had marked both the first papers, or both the second papers, present so that the examining team could be confident that all candidates, irrespective of the papers written, were awarded the correct grade.

It was clear that there was a difference between the time zones with respect to the Internal Assessment. As a consequence a number of candidates had their final grade reduced by one. Candidates must be trained and encouraged to write full, comprehensive, original, and relevant internal assessment projects.

It was also evident that both first papers did not have a time issue whereas both second papers did. There were two graph drawing questions on each second paper and the candidates found it difficult, under examination conditions, to complete these and answer all the other questions fully. This concern was taken into account during the grade award process.

Standard level project

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 4	5 - 6	7 - 8	9 - 11	12 - 14	15 - 16	17 - 20

It was noticeable this session that the mark awarded for the project did make a difference to the candidate's overall grade. Many projects appeared to be rushed, were far too short, contained many errors and obviously did not even use the 20 hours class time that should be allocated to the project. As a result a poor attempt at the project had the effect of lowering the candidate's grade by one point. Other candidates had taken some pride in their work and produced very good pieces of work that had been carefully checked by their teacher. A good project generally increased the overall grade by one point.

Range and suitability of work submitted

Almost all the tasks chosen this session were appropriate for a Math Studies projects. Only in some cases were the topics too descriptive having little or no mathematical content in them. There were also isolated cases of what appeared to be an attempt at the mass production of projects by individual schools where all the projects had a similar problem to solve and the candidates solved it in the same way. Candidates attempted projects from a variety of branches of mathematics and it was really refreshing to moderate these. However, projects with a statistical basis still predominate. A few candidates used statistical tests that were outside the syllabus and, even when the mathematics was accurate, it appeared that they did not always fully understand the tests they were using.

The majority of the projects were well presented with few this year being hand written. There were a number of very short projects and a number of unfinished projects. The work produced needs to reflect the level of the course. Also, the project must not simply be a piece of work with loads of calculations. There must be a flow of ideas between the different sections and the interpretations of the results generated need to be clear and concise. Graphs must be clearly labelled, pages numbered and all sources documented. A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the

calculations. Many candidates did not show any simple mathematical processes this session and went straight to sophisticated mathematics. Some of the candidates did not have any calculations at all and just showed results from their calculator or computer with formulae nowhere to be found. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

A few candidates are still downloading “their project” from the internet. Teachers should be monitoring the project during the various phases to avoid cases of plagiarism. When using the internet the candidate must remember to include the web address in their bibliography.

Candidate performance against the criteria

A. Most of the projects had a title this session. The majority of the candidates stated their task but there are still some candidates who find it difficult to explain this in a clear and concise way. In most cases this occurs when the topics chosen unsuitable and should have been discouraged by the teacher. Most of the candidates explained how they were going to do to collect their data, but not all of them described the mathematical techniques they were going to use in their project. Not all the plans were well focused. Candidates with clear statements of task and plan generally wrote more successful pieces of work.

B. The data collected was generally relevant to the stated task. On the whole it was well collected and well organized ready for analysis. In general there was no problem with the quantity of data but the quality was questionable in many cases. A few candidates did not include their raw data so it was difficult to see how this had been structured ready for analysis. Some candidates forgot to include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given making it impossible for the moderator to check whether or not the tables of data were accurate. A large number of candidates just downloaded data straight from the internet with little thought being given to how much of that information was really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way. It is also important to state the website in the bibliography.

C. The mathematics used needs to be done in a meaningful manner. Some projects contained many mathematical calculations, some of which were meaningless for the actual project. The importance of relevance must be stressed. When a scatter diagram indicates that there is no correlation between two variables then it is meaningless to go on to calculate the correlation coefficient and the line of best fit. Working out standard deviations without a meaningful discussion on what the results

indicate is of no value. Some candidates are applying sophisticated techniques in their analysis and are omitting the simple mathematics and/or the use of graphs to analyze their information. Many candidates relied entirely on computer generated results and no explanations of the techniques used appeared in their project. With some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. A growing number of candidates and teachers do not realize that no more than 20% of the expected cells can have values less than 5 and none of them should have values less than 1 for the test to be valid. Also several candidates used raw data instead of frequencies.

D. Almost all the candidates were able to produce one conclusion or interpretation that was consistent with their analysis but often these were rather brief. In a many cases the conclusions were just a one or two line statement. This does not produce a meaningful discussion of results but rather a series of disjointed remarks. Teachers need to encourage candidates to ensure that their interpretations and/or conclusions are developed in a comprehensive way. Sometimes candidates were unable to discuss their interpretations/results through their own lack of understanding of the relevance of the process used.

E. Validity was generally attempted by most candidates although very few achieve level 2 for this criterion. The majority of the candidates were successfully able to comment on their data collection method or their interpretations and/or conclusions but few commented on the mathematical processes that they had used or suggested possible means to improve their project.

F. The overall presentation of the project work was good. The majority of the projects are now word processed, hence easier to read with tables and graphs clear to follow. In most cases there was an attempt to structure the work but in some cases the work was not sufficiently linked together. In some projects questionnaires used for surveys were not included and in others the data was either missing, not set up for use or was relegated to an appendix. In most projects correct mathematical language was used. Many candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations and guidance for future teaching

Teachers can help their candidates in many ways:

- Write clear and full comments on the 5/PJCS forms.

- Discuss a detailed plan of the project with the candidate to see if the task chosen has a mathematical future.
- Encourage candidates to work on the evaluation area of their project in more depth.
- Encourage candidates to use a wider variety of mathematical techniques both simple and sophisticated.
- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.
- Emphasize the importance of showing, where appropriate, sample calculations in both simple mathematical processes and sophisticated techniques.
- Stress the importance of using appropriate notation and terminology.
- Stress the importance of documenting sources. Where does the data being analysed come from.
- Discuss in class the significance and limitations of specific techniques.
- Remind candidates that their value for r or the chi-squared test statistic only supplies evidence for or against a relationship, which can be strong but is never proof.
- Give candidates a second chance to correct errors.
- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Tell them to include all raw data – but not <u>all</u> the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.

- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them.
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.

To improve the security of IB examinations, a selection of examination papers now have regional variants, including Mathematical Studies papers 1 and 2. The following report is for Mathematical Studies taken by candidates in the IB regions of Africa, Europe, the Middle East and Asia/Pacific and also for those candidates taking the examination in Spanish and French.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 22	23 - 33	34 - 46	47 - 59	60 - 72	73 - 90

General Comments

It was difficult for candidates to remember to put units in every time, give answers to 3 significant figures every time and answer financial questions with the correct number of figures as stated in the question. This meant that many candidates lost at least 1 or 2 marks.

The areas of the programme and examination which appeared difficult for candidates

The questions which students consistently found difficult were:

- Drawing the full frequency polygon – i.e. both starting at ½ and 5 ½ and joining the mid points
- Distinguishing between rational, integer and positive integer numbers
- Finding statistical values without realizing the meaning of them by just relying on the calculator's value i.e. a mean value in Qu 5 of 127 etc.
- Interpreting a trig equation and then applying it
- The concept of domain and range in Qu. 10.
- Interpreting questions especially Qu. 15.

The areas of the programme and examination in which candidates appeared well prepared

Most schools appeared to have covered the syllabus and the majority of candidates were able to make serious attempts at all the questions. Good working was shown by many (but not all) of the candidates so that follow through marks and method marks could be awarded when parts of questions were incorrect but working was seen.

Logic, linear equations, sets, currency conversions, differentiation and probability were well answered by the majority of the candidates.

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Logic

This question was well answered by many of the candidates. It is an area of the syllabus that is well taught and many managed to get a follow through mark even though one of the columns in the table might have been incorrect.

Question 2: Frequency Histogram and statistics

Very few candidates could draw a frequency polygon correctly. The word 'Draw' means that a ruler should be used. Many managed to draw from the mid-point of the bar but did not extend it to 0.5 or 5.5. Most could answer the probability part of the question.

Question 3: Sketching a diagram and Trigonometry

The initial diagram was well drawn by most candidates but few could extend AC to find D. The point D was either drawn between A and C or on CA extended. When on CA extended the candidates could be awarded A1 follow through for the angle. A surprising number of candidates could not find the correct answer for the length of BC.

Question 4: Number Sets

This question was poorly answered with many thinking Q was the set of irrational numbers. Very few candidates were awarded full marks for this question.

Question 5: Statistics

Many candidates did not use the calculator correctly to find the mean and standard deviation, trying to do all the calculations by hand with a significant number not realizing it was a frequency table – this could have been the error with those using the calculator also as few received full marks on this question. When the candidate had made an error follow through marks could be awarded for the final part *provided working had been shown*. Most knew the data was discrete.

Question 6: Linear Equations and Solving Simultaneous Equations

This question was well answered by some candidates and poorly answered by others. It seemed to be part of the syllabus that might have been fully taught by some schools and not by others. It was surprising to see how many candidates could not find the gradient of a perpendicular line when this has been tested for many years.

Question 7: Sine Function

This was the most difficult question on the paper and proved to be a good discriminator. Many good candidates gained full marks but the majority were unable to interpret this question in terms of real life and could only find the amplitude.

Question 8: Compound Interest

Again this is a question that has been tested before but few candidates managed to gain full marks. Many, in part (b), believed they had to subtract 8000 from the value to get the interest first. This could possibly be a result of the way the formula is given in the formula booklet so teachers need to be aware of this.

Question 9: Sets

This question proved to be one of the easier questions with a number of candidates able to shade in the required region and finding values in a set. They still had problems with part (b).

Question 10: Factorising, Domain and Range

It was surprising how many candidates could not factorise this expression. Of those that could some went on to find the zeros of a quadratic equation which was not what the question was asking. Some confused domain and range and many did not write down all the values when they did know domain and range.

Question 11: Currency Conversions

This question was well answered by a number of candidates with few confusing the conversions. Some found the last part difficult with many leaving it out.

Question 12: Differentiation Equation of the Tangent

The final part of this question was not well answered. Most candidates could gain 4 marks in this question as most knew how to differentiate and they were required to do it twice. However, few realized that they could find the gradient of the tangent from their answer to part (a). This part was badly answered by most candidates.

Question 13: Cumulative Frequency and Box and Whisker Plot

This was done well by most candidates. Some did not realise what the inter-quartile range was, and just quoted the quartiles.

Question 14: Probability

This question proved to be the easiest question (along with question 1) with many candidates gaining full marks. The probability tree diagram was completed correctly and then most could go on to find the required probability. Very few added the probabilities instead of multiplying them.

Question 15: Calculus

This question was poorly answered by many of the candidates. They could not write down the equation of the tangent, they could not say whether one value was greater or less than another and they could not answer that P was a minimum point. Most attempted the question so it was not a case that the paper was too long. This was a very good discriminator for the paper.

Recommendations and guidance for the teaching of future candidates

Teachers need to make sure all candidates can use a GDC for statistical calculations including mean and standard deviation, correlation coefficient and chi-squared.

Apart from obviously teaching the syllabus thoroughly, teachers should be teaching examination techniques such as

- Does the answer make sense or is it reasonable?
- Read the question before beginning to answer the question
- Show working for all problems.
- Teachers also need to point out to candidates that unit penalties, financial penalties have also been added to the original accuracy penalty.

Further Comments

There were some schools where most of the candidates did not achieve 50% of the total 90 marks on the paper. It was clear that they had not been taught the full syllabus.

Standard level paper two

Component Grade Boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 9	10 - 19	20 - 28	29 - 40	41 - 53	54 - 65	66 - 90

General Comments

This paper differentiated the candidates as the range of marks ranged from full marks to very few marks. However, the distribution was normal with a slight lean towards the left. Time seemed to be a factor as many candidates did not reach the last question of the paper. Comments from the G2 supported the idea that the time problem was further increased by the fact that there were two graphs on the paper. After some considerations of the teachers and examiners' comments, and also exposure to many scripts, the senior team came to the conclusion that no individual question was unacceptable and this was clearly reflected in the G2 forms with the majority of teachers considering the paper as appropriate. However, there was a combination of factors that resulted in many not completing the paper. These factors are highlighted in the details for each question below.

Although many candidates appeared to be well prepared as they gave their answers to the correct number of significant figures or to the specified accuracy in the financial questions where appropriate, and wrote down their answers with their corresponding unit alongside, many others were not aware of the need to do this and were penalized with the three penalties, accuracy, unit and financial leading to the loss of three marks in the paper.

In general graphs were well done though some candidates lost one mark for not labelling the axes.

The areas of the programme and examination which appeared difficult for candidates

- Recognizing the need for AP and GP formulae and distinguishing between them.
- Using simple interest formula
- Knowing what to do in ¨show that¨ type questions
- Conditional probability
- 3D trigonometry
- Finding the equation of the horizontal asymptote
- Using the derivative for optimization and justifying the results
- Timing the exam

The areas of the programme and examination in which candidates appeared well prepared

- Graphs were mostly well done. Plotting points was mostly accurate.

- Simple probabilities were well calculated
- Working was in general shown. Very few candidates wrote down their answers without showing their working. This was an improvement on previous sessions.
- General ideas of correlation and the chi- squared test were well understood
- Use of trigonometric ratios was done well. Especially the use of the sine and cosine rules

The strengths and weaknesses of candidates in the treatment of individual questions

Question 1: Sequences and series - Financial maths

This question was answered correctly by many. Candidates were able to restart if they failed to complete a particular part. Many candidates wasted much time because their understanding was limited to a recursive method and hence wrote out all the terms rather than using the formula for the nth term or sum. A surprising number of students were not able to use the simple interest formula for a period which was not a whole number of years. Also hardly anyone knew to calculate interest first before substituting into the formula. Many students who attempted part (d) lost a point due to FP. A number of students rounded their answers prematurely to the nearest dollar.

Question 2:

(i) Chi-square test - Probability

Candidates answered part (a) correctly. Some lost one out of the 4 marks for making an error in the denominator of the conditional probability. In (b) many students failed to see that (b) was ´without replacement`. Parts (c), (d) and (e) seemed to be very well done by some centres and uniformly badly by others. In (e) many gave the table from the GDC and highlighted the value 63 for which no mark was gained. Expected value formula should have been used instead.

(ii) Linear correlation

The graph was well done with almost all candidates labelling and scaling the axes correctly. A minority of students joined the points or drew the graph on lined paper which prevented them from gaining full marks in this part of the question.

In (b) some candidates were not able to calculate the linear correlation coefficient. A few G2 comments pointed out that the command term used may have been ambiguous to some candidates and they did not think that they could use their GDC to find r. Some attempted to use the formula even though the value of S_{xy} was not given. The guide says that ' A GDC can be used to calculate r when raw data is given'. This potential unfairness was taken into

consideration during the setting of boundaries so that no candidate was disadvantaged by the possible ambiguous wording of the question. In future the command term 'Using your GDC' or 'Write down' will be used in similar questions.

Some students who did use the GDC gave r^2 instead of r. This really caught the attention of many examiners as r^2 is not in the syllabus.

Question 3

Many students did not write the units in their answers and were penalized with the UP in this question.

(i) Trigonometry

Part (a) was not very well answered. It looked as if the candidates did not understand the question. Many candidates did not draw a sketch of the triangle; this would have helped them to solve the question. Many candidates simply calculated the remaining angles of the triangle and showed that the sum was 180^0. This was a clear example of the misunderstanding of the term ¨show that¨. Part (b) was well done though some candidates lost a mark for not giving the answer to the correct accuracy.

(ii) Geometry of three-dimensional shapes

The weaker candidates spent a lot of time in (a) using the wrong triangle to find half of the diagonal of the base. Finally they used Pythagoras theorem with the wrong numbers. Part (b) was well answered by most of the students. For the volume of the pyramid in (c) they used the correct formula though not always with the correct substitutions. To find the height of the prism in (d) the most common error was multiplying the volume of the prism by $\frac{1}{3}$. It seemed that many did not know the term ´prism`.

Question 4

(i) Functions

There were many well drawn graphs using correctly scaled and labelled axes with a good curve drawn. A number of students did not label the maximum point. Although many students showed in their graph the asymptotic behaviour of the curve, they did not know how to describe the asymptote. It was noticed that some students were tracing the curve to find the coordinates of the maximum instead of finding the maximum directly. The intersection between the line y=1 and the curve was not always read from their graph but from their GDC´s graph.

(ii) Calculus

Finding the derivative was done at least partially correctly by most of the candidates. However, using it to find the minimum and to justify why it is a minimum was troublesome for

the majority of the candidates. Even those who used a graph in their reasoning neglected to mention the change from decreasing to increasing or to supply a sign diagram. Many candidates recovered in the last part of the question when finding the minimum cost.

Question 5: Number and Algebra - Geometry

A number of candidates did not attempt this question worth 12 marks but the majority answered this question partially and were able to gain some marks. Parts (a) and (b) were mostly well done. Very few candidates managed to answer part (c) well; this part of the question required good algebra along with a clear understanding of the situation given in the diagram. Many recovered then in (d) when they were asked to write down the quadratic equation. Solving the equation was not always found to be easy. Use of the GDC was expected but many used the formula. The correct solution, t=3, was chosen in the last part of the question. However, their justification was often false causing them to lose both the reasoning and the answer mark.

Recommendations and guidance for the teaching of future candidates

- Train to know when the use of the GDC is appropriate, e.g. chi-squared and regression correlation. This will also leave them with more time to spend on other questions.
- Thoroughly familiarise them with their formulae and when to use them so they recognise the need quickly and go straight for the correct one.
- Teach better understanding of what is needed to ´show that`.
- Applying AP´s, UP´s and FP´s should be part of a teacher's routine in marking throughout the 2 year course.
- The use of sketches is an integral method in trigonometry which will help many candidates to come to terms with what they are being asked to do.
- Remind students to label and scale the axes each time they draw a graph
- Students need to monitor their time more carefully and need more guidance on how to pace themselves during this 90 minute exam (1 mark per minute).
- Teach them the importance of getting an answer that corresponds to the question asked, consider the result and apply common sense for errors and misunderstandings