MATHEMATICAL STUDIES

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-28	29-39	40-53	54-67	68-80	81-100

This was the first examination for the new curriculum and hence also the first with options absent from paper two. It was felt from early on that the overall impression was weighted towards the harder end of the scale. Having no previous sessions with which to compare directly, the grade award process was demanding and took even more deliberation than usual, but all participants felt that it was about right when the final decisions were imparted. Allowance was made in places for question parts that were particularly difficult and also for a couple of places where misunderstanding of the wording was deemed possible. These were minor flaws and the session was without any major problems.

Internal assessment boundaries are set for the whole period of the current curriculum. This was done initially at the end of April after some early marking of projects but was reviewed at the grade award meeting and the decision was adjusted at one boundary level to react to subsequent marking experiences.

Standard level internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-8	9-11	12-14	15-16	17-20

The range and suitability of the work submitted

This is the first session in which the new criteria were being used to mark the projects and it appeared that the majority of teachers were using them successfully to grade the projects. Surprisingly, some teachers were still using the old criteria. Most of the projects this session were appropriate, if lacking sometimes in originality and content. In some projects the tasks chosen were too narrow but in most cases the candidates did have sufficient scope to demonstrate their mathematical ability. Candidates attempted projects from a variety of branches of mathematics and it was really refreshing to moderate these. However, projects with a statistical basis still predominate.

Most of the projects were well presented with few this year being hand written. There were a number of very short projects. The internal assessment is meant to be a substantial piece of work and three or four pages of simple mathematics will not score highly in a number of criterion areas. A number of projects did not contain the raw data. This makes it impossible for the moderator to check the accuracy of the calculations.

There was a significant increase in the number of candidates using the chi-squared test and linear regression this session. A major concern is the number of candidates and teachers who fail to realize that no more than 20% of the expected cells should have a number less than 5 and that no expected cells should have a number less than 1 for the test to be valid. Also there is no point in finding a correlation coefficient or regression line if a scatter diagram has shown that there is no correlation.

More candidates are now using their GDC to do the mathematics for them and often they forget to write down the formula they are using and mention why a particular procedure is being used. This has the result of leaving the moderator to wonder whether or not the candidate really understands what they are doing.

When using the Internet the candidate must remember to include the web address in their bibliography.

Candidate performance against each criterion

A. Most of the topics chosen were appropriate for a Mathematical Studies project. The majority of the projects had a title this session. There are still some candidates who find it difficult to explain in a clear and concise way their statement of task. In most cases this occurs when the topics chosen were quite difficult to understand. When describing the plan, many candidates explain what they are going to do to collect their data, but only some of them describe the mathematical techniques they are going to use in their project. Candidates with clear statements of task and plan tended to be able to extract more depth from their projects because they knew what they were looking for.

B. The data collected was generally of sufficient quantity but was not always focused on the task. It was easier to find projects where the data could be considered enough in quantity but not in quality. A few candidates did not include raw data within their project or as an appendix, while some failed to include a sample questionnaire if this was the method they used to collect data. In these cases only final tables of data were given. It is very difficult for the moderator to check accuracy in cases like this. A large number of candidates simply "dump" tables and charts straight from the Internet into their project, with little thought being given to how much of that information is really relevant to their task. The organization and presentation of relevant data becomes crucial when data is collected in this way.

C. Most candidates used basic mathematical techniques for analysis, many relying entirely on computer generated results. Many of these candidates omitted explanations and clarifications of these techniques and were not selective about using the particular results that were relevant for their investigations. Some candidates are applying sophisticated techniques in their analysis and are omitting the simple mathematics and/or the use of graphs to analyze their information. As previously mentioned, with some of the statistical techniques, like the chi-squared test, it was evident that not all candidates knew what they were doing. Also, the mathematics needs to be done in a meaningful manner. Some projects contained many mathematical calculations, some of which were not relevant for the actual project. The teachers differed in their interpretation of what constituted "sophisticated" mathematical techniques and this was an area that often required moderation.

D. Almost all the candidates were able to produce conclusions or interpretations that were consistent with their analysis but sometimes these were rather brief. In a high number of cases the conclusions were obvious and not very thorough.

E. More candidates commented on validity. Usually this was more to do with the data collection than anything else. A few commented on the mathematical processes that they had used. Of those who did, few reached the level of thoroughness required for a high level of achievement.

F. Although in a few cases questionnaires used for surveys were not included in the project and in others it was difficult to follow the process because important data had not been set up for use or had been relegated to an appendix, on the whole, projects were easy to read and well structured. In most projects correct mathematical language was found and the tendency to repeat calculations and techniques on different sets of data, coming up with pages and pages of repetitive and often needless information has diminished to a certain extent. Many

candidates now include a bibliography and references to sites accessed, although the latter is not always well documented.

G. The majority of the teachers appear to have awarded marks appropriately.

Recommendations for the teaching of future candidates

Teachers can help their candidates in many ways:

- Encourage candidates to work on the evaluation area of their project in more depth.
- Encourage candidates to organize the data they collect in ways that makes it easier for the reader to understand how it is to be used in the development of the project.
- Emphasize the importance of showing sample calculations in both simple mathematical processes and sophisticated techniques and to present those calculations, irrespective of the use of technology.
- Assist in the selection of topics and discourage topics that are too narrow or one-dimensional.
- Stress the significance of collecting sufficient data to perform certain techniques.
- Encourage candidates to comment on the procedures they are going to use and reflect upon them once completed.
- Give them examples of "good" projects so that they know what is expected of them.
- Encourage class discussion on factors that affect the validity of questionnaire data.
- Make sure that they are aware of (and understand) the assessment criteria.
- Encourage them to think up their own task and explain the plan thoroughly.
- Tell them to include all raw data – but not all the completed questionnaires! A sample is sufficient as long as they gather all the data in organized tables.
- Check that the mathematics used in the project is relevant.
- Encourage the candidates to use more sophisticated mathematics.
- Pay special attention to the validity demands of the chi-squared (or other) test and spell out in detail the null and alternative hypotheses and degrees of freedom.
- Explain to the candidates how to evaluate their work, draw conclusions, examine the mathematical processes used and comment critically on them
- Send the original work of the candidate to the moderator.
- Meet with the candidates at regular intervals to monitor the progress of the project.

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-12	13-24	25-37	38-49	50-62	63-74	75-90

General comments

After careful consideration of candidates' performance and diligent reading of comments by teachers and examiners, it was concluded that this paper one was at the more difficult end of the acceptable spectrum. In fact this should be about as hard as any paper one ever gets. The lower than usual grade boundaries reflect this judgment. There were one or two places in this paper where teachers complained about lack of clarity and possible difficulty for candidates working in a second language. These comments were taken seriously at standardisation and measures were taken to protect such candidates.

Having said all this, there were still very many positive comments and the overall impression conveyed by questionnaires was that the paper was within acceptable bounds.

There were still plenty of good performances and not an unusual number of poorer efforts. Time was a problem for some, though again this was only slightly more noticeable than usual.

The areas of the programme and examination that appeared difficult for the candidates

The question on probability was by far the worst handled. The examining team was shocked at how poorly this question was treated. There seemed to be almost no understanding at all of non-replacement.

Many candidates were not fully conversant with the power of their GDC and did not automatically turn to it for help in all the appropriate places.

The process of finding the second derivative was not well understood and negative powers caused confusion in this context.

The containment structure of the basic number sets was not well known.

Detailed investigation of the number of members in different parts of a Venn diagram was rather unreliable. Percentage error was poorly done. It came to the attention of the examining team that a number of schools were using an old version of the formula booklet, which contained an incorrect formula for the percentage error (it gave the formula for absolute percentage error). Use of the absolute value bars by a candidate indicated this and so markers were told to allow this formula for this session if all else was correct. In future though it will be assumed that schools are using the most recent formula booklet.

The areas of the programme and examination in which candidates appeared well prepared

Most schools appeared to have covered the syllabus completely, though there were a few cases where it was obvious that this had not happened. Basic statistics and boxplots were well done. Basic currency calculations were handled quite well. Somewhat surprisingly, the comparison of cubic function shapes was quite well done, though a small contingent had no clue about this.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Number Sets.

Quite well done, though there were several common errors. 1 and -1 were interchanged sometimes, the recurring decimal was considered non-rational. Some teachers expressed concern about the notation for this decimal, but in practice very few candidates had problems with it. Nevertheless the concern has been noted. Some candidates misunderstood the requirement of the question and entered each number multiply in each set to which it belongs, ignoring the subset containment already implied by the diagram.

A follow-through scheme was devised for this situation to avoid severe multiple penalties.

Question 2 Discrete Stats and Boxplot.

This question was done well overall. Some candidates did not use the scale on the paper for their diagram. The value of the range should be a single number, not an interval, though the latter was treated leniently.

Question 3 Significant Figures, % Error.

Parts (a) and (b) were handled OK. The percentage error was not so good though. Allowance was made here for use of old, incorrect formula book, however this will **not** be repeated.

Question 4 Logic statements, Truth Table.

This question was quite well done**.**

Question 5 Frequency Histogram.

Some candidates found this question confusing. Rounding and boundaries of the regions were not well understood.

Question 6 Plotting Points, Trigonometry.

Surprisingly, this was not done well. Many candidates assumed a right-angled triangle was involved and simplified all their formulae inappropriately. Some were careless in plotting the point.

Question 7 Quadratic Function.

Some managed this OK. It was accepted that the wording could have confused many candidates and alternative interpretations in part (c) were allowed if they seemed reasonable.

Question 8 Probability.

Very poorly done, the worst question for almost all candidates. There was almost no understanding of how to deal with the no replacement property.

Question 9 Cosine Rule, Geometry.

The slightly unfamiliar form of this question perplexed some candidates. Despite that, the overall performance was not bad.

Question 10 Currency Conversion.

Most candidates did this well, with only a few reversing the rates and/or processes. Most also got at least part marks for part (c).

Question 11 Simultaneous Linear Equations, GDC Solution.

Much time was wasted here trying to solve the equations on paper, by hand. Due to the nature of the equations, this also often went wrong. This was clearly a question to be solved with the GDC.

Question 12 Properties of Functions, Vertical and Horizontal Shift.

This was **not** intended as a question on cubics. Rather it sought to test understanding of basic function properties, when shifting vertically or horizontally. It is accepted that the question contained a misunderstanding of the word 'increasing'. Examiners were instructed to mark 'all graphs increasing' as correct, as well as the answer A.

Question 13 Numbers in a Venn Diagram.

Poorly attempted. Candidates often failed to understand that an expression containing x was required for part (a).

Question 14 Differentiation.

As might have been predicted, differentiation of negative powers caused problems for many. Some candidates only reached the first derivative. Many did not recognise the simple need for a substitution as answer to part (b).

Question 15 Using GDC to Plot Unfamiliar Functions.

The question wanted to test use of the GDC to sketch functions that would be unfamiliar to these candidates. Many candidates had a go at this and some were very successful. However, common errors included omission of the brackets round the denominator. Follow-through was allowed for this later in the question. There were some excellent sketches, but clearly more guidance is needed in many schools about what is required for a sketch, the mark-scheme for this question discusses this to some extent. Axis labels were desirable, but not essential. Some indication of scale is needed, if only to act as guidance for the approximate position of important points such as intersections. In this sketch, a tick at 1 on the vertical axis was appropriate. The rational function needed to have clear horizontal asymptotes at the x-axis and the root needed to increase reasonably and not become flat or too vertical. The intersection was expected to be roughly in the right position.

Recommendations and guidance for the teaching of future candidates

Some of the following advice has been pasted from a previous report, as it remains relevant and important. Regardless of any other factors, the course must be covered in full and candidates need to practice the work sufficiently to be able to cope with variations of the context contained in any examination paper. Candidates need to be exposed to a wide variety of problems within each topic to ensure that they can establish appropriate problem-solving skills.

Candidates would be well advised to consider how parts of questions lead on from each other.
Candidates should to be taught how to use their graphical display calculators properly and how to write their working when using the GDC. They need to gain confidence in resorting to the GDC when that is appropriate and in recognizing the need for this.

It is important to check that your school is using the most recent version of the formula booklet. This can be ascertained by checking the % error formula. This should NOT contain an absolute value notation.

Of all the topics in the syllabus, the one which seemed weakest in paper 1 this session was the probability. Much more practice is needed here.

Despite the paper being towards the harder end of the spectrum, there was much good work seen, and this is to be commended, but can be further enhanced by regarding the advice and comments in this report.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-12	13-25	26-33	34-46	47-58	59-71	72-90

General comments

Candidates generally found this paper difficult and long. The G2 comments from teachers were generally favourable with more than 85% stating that the level of difficulty was appropriate with almost all teachers agreeing that the syllabus coverage, clarity of wording and the overall presentation of the paper was either satisfactory or good. Many teachers passed on comments from their students who thought that the paper was fair but too long.

With graphic display calculators now compulsory in the Mathematical Studies course, it had been expected that many answers would have been found using technology. In practice this did not necessarily happen, with many candidates spending a lot of time writing out detailed algebraic solutions to questions and hence not having time to complete the paper. This applied to both paper 1 and paper 2. Candidates should be fully prepared in the operation of their calculators, especially in those areas of the syllabus where their use has been identified.

The areas of the programme and examination that appeared difficult for the candidates

Examples of the lack of use of the graphic display calculator were clearly identified in questions 4 and 5. In question 4, it was expected that the substituted formula would be written down and then the answer found using either the 'solver' or 'tvm' software available. A significant number of candidates tried to use logarithms to solve the first part and ended up in all sorts of bother.

In question 5, candidates were asked to write down the chi-squared statistic and it was expected here that they would read this value from their calculators. Many candidates attempted to calculate this statistic longhand, making errors and losing valuable time. Similarly, in the second part of question 5, the correlation coefficient and regression line values should have been found via the calculator. There was not enough information given in the question for candidates to apply the relevant formulas.

Candidate performance in both questions 4 and 5 was generally poor. In the first case it was due mainly to a lack of knowledge about periodic (monthly) compounds and limited ability to 'see' the term of a geometric sequence. The wordiness of question 4 may have caused concern for some. In the latter case, time was definitely a factor.

Many candidates struggled in parts of questions 2 and 3, due mainly to a lack of knowledge or practice with number theory and calculus. Candidates should expect an examination to cover more than just the basic examples and many seemed unprepared for something a little bit different.

There was a general weakness shown by candidates in attempting "show that" type questions and with giving reasons for results.

The areas of the programme and examination in which candidates appeared well prepared

All questions in the paper were well answered at least some of the time indicating that the paper was accessible. Many candidates had been thoroughly prepared, demonstrating sound knowledge of the whole syllabus and the ability to apply their knowledge across a range of problem solving areas.

Accurate graphing of a quadratic function (q1), Pythagoras and non-right angled triangle trigonometry (q2), Simple polynomial differentiation (q3) and the Chi-squared test and linear regression (q5) were all answered quite well.

At the same time, there were a large number of candidates who demonstrated only partial knowledge of the course and very limited ability in its application. Many questions were left unanswered by students with comments such as "we have not studied this" added to their scripts.

Many candidates were able to gain marks by working neatly and logically and by showing clearly the method they used to solve problems. This aspect of examination technique has improved over time.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 Quadratic and linear functions and, in particular, accurate graph drawing, knowledge of domains, increasing functions and points of intersection.

Candidates generally drew good quality diagrams though a significant number left off the labels and, in some cases, the scales. Candidates were able to find and plot the points from the table and most drew a neat smooth curve. In many cases a mark was lost because the curve was drawn outside of the given domain and/or arrows added to the ends of the curve. A similar comment applies to the linear function which should have been drawn in the first quadrant only according to its given domain.

A number of candidates found it difficult to show that the vertex of the function had particular coordinates though many saved themselves to some degree by showing method lines on their graph. Very few candidates gained full marks for knowing where the quadratic function was increasing which was disappointing. It is clearly specified in the syllabus.

The final parts, asking for the point(s) of intersection and vertical distance were done quite well.

Some concern raised by teachers in their G2's in regard to the given domain and point(s) of intersection were covered by allowances in the markscheme.

Question 2 Sets, Probability and 3-D Trigonometry.

In general, this question was poorly done though there were many candidates who gained close to full marks.

Most candidates were able to list set A, but that was it for some. It was obvious that a significant number of candidates had little idea about number and sets. It was common for the intersection of A and B to be given rather than the union and there were many wild guesses about the complement of set C. Some students were able to gain follow through marks by listing each of the sets they were dealing with - a very sound practice.

The probability part was most disappointing. Many candidates listed 21 as the denominator not realising that it was supposed to be the number of elements in set A.

The first part of the second section was answered well with most candidates able to apply the cosine rule to find the required angle. Many were also able to show that the surface area of the prism had the given value.

The final part was poorly answered. G2 comments suggest that the diagram was misleading and it was difficult to visualise the triangular prism. This is possibly true as many candidates attempted to find the volume of a tetrahedron and not a prism. It was also suggested in the G2's that a diagram of the triangular prism should have been given but this would have defeated the purpose of the question, which was, in part, to test knowledge of planes. In setting grade boundaries, this potentially unfair difficulty was taken into consideration.

Question 3 Calculus and Coordinate Geometry.

This question was well answered by many candidates and showed clearly the distinction between those who have been well prepared and those who have not.

There was some concern expressed about the use of fractional coefficients but this turned out to be not well founded. Most candidates were able to find g(2) and the first derivative and thus gain 5 marks. However, many students fell by the wayside at this point, using the function and not the first derivative to show that the gradient was 8 and being unable to find the equations of the tangents.

The sketch of the function and tangents was generally well done. Most candidates chose a suitable window and provided a neatly drawn sketch. Many candidates, who could not find the equations of the tangents in the earlier parts, could at least provide the horizontal tangent on their sketch. Some candidates did not draw the straight lines as tangents but as non-touching lines somewhere on their sketch.

A sketch does not need to be done on graph paper and scales do not have to be perfectly accurate as long as the important features of the shape are demonstrated.

Question 4 Finance and Geometric Sequences.

This question proved the most difficult for candidates. Many candidates seemed to be unfamiliar with periodic compounds and used annual interest throughout the question. They were able to gain many follow through marks if the substituted formula was shown. It was clear that a significant number of candidates are unfamiliar with the TVM type software available with the graphic calculators. Much time was lost by candidates in the early parts of this question by writing long solutions to the problems. This included the use of logarithms in part (i). A correctly substituted formula and the answer is sufficient to gain full marks. For the simple interest part (b)(ii), many candidates forgot to subtract the capital to find the interest. They lost one mark.

The second part of the question was poorly done with many blanks appearing in the scripts. It was treated more as a compound interest question rather than a geometric sequence with the exponent of 5 and not 5-1 being used. Many candidates calculated values for each year rather than equate the two functions. This was quite time consuming and again, caused difficulty in completing the paper.

Candidates who persisted with this question often did gain good follow through marks despite making errors in a number of places.

Question 5 Chi-squared test and Linear Regression

Unfortunately, many candidates were struggling for time to answer this question. For those who did so, the responses were quite good. It was expected that graphic calculators would be used throughout this question but, as noted earlier, this did not necessarily occur. Many candidates attempted to find the chi-squared statistic and both the correlation coefficient and regression line values by use of

formulae. This was both difficult and time consuming. The allocated marks should have provided the candidates with a clue that minimal working was expected here.

Some candidates did not write the expected values as integers but otherwise, the first part was done quite well. Many candidates failed to give the appropriate reason for their test decision which was simply that the calculated value was greater than or less than the given critical value. Both marks were lost if the correct reason was not written down.

Those candidates who had time to complete the second part of this question gained good marks. The majority of candidates lost one mark because they were unable to explain what a positive value for the coefficient of correlation meant and quite a few candidates could not write down a sensible answer for why the answer to a value which had been extrapolated may be unreliable, however, examiners were instructed to accept alternative answers here if reasonable.

Recommendations and guidance for the teaching of future candidates

It was pleasing to note that the majority of candidates present scripts with good, neat working shown. Unfortunately, the basic examination technique of some candidates remains poor. They are advised to start each question on a new page and yet many squash their work up to such a degree that it becomes difficult to mark, particularly in trying to award follow through marks. Students are asked to write in pen and yet many do not, presenting light, difficult to read work.

In general, graphs and diagrams were well drawn though many candidates forget to label and scale their work. The use of diagrams to illustrate and assist problem solving is to be encouraged.

The graphics calculator is compulsory in this subject. Full marks will be awarded for a solution which contains an indication of method as well as the answer. The method shown should be in mathematical terminology, not that of the calculator, when these differ. Candidates will continue to struggle with the length of the examination if they persist in writing out full algebraic solutions when a GDC solution is acceptable. Teachers must inform candidates of how the graphics calculator can assist in solving problems and teach the use of command terms which include "write down", an indicator that working is not required.

Further comments

It is quite noticeable that many candidates have not been exposed to the whole syllabus and others have had little opportunity to practice the material in any depth.

Candidates should have as much practice as possible in answering questions written in different styles. It is clear that many candidates are nonplussed when confronted by questions in an unfamiliar format.

Candidates should have access to questions which ask for reasons and questions which require them to show that a certain result is true. They should clearly understand what is required in such questions. Too often, these were left blank in this examination or, in a number of cases, a paragraph of non-mathematical gobbledygook presented.

