May 2015 subject reports

Further Mathematics

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 11	12 - 23	24 - 36	37 - 48	49 - 60	61 - 72	73 - 100

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 34	35 - 54	55 - 70	71 - 87	88 - 103	104 - 150

General comments

The areas of the programme and examination which appeared difficult for the candidates

Many candidates are unable even to attempt questions on pure geometry.

Many candidates do not make efficient use of their calculators. They need to be aware that many of the problems on statistical inference can be solved completely using the statistical software on the calculator.

Many candidates seemed to be unfamiliar with some aspects of linear algebra.

The areas of the programme and examination in which candidates appeared well prepared

In general, candidates are well prepared to answer questions on groups, properties of graphs and number bases.

The strengths and weaknesses of the candidates in the treatment of individual questions

The candidature was disappointing with a number of excellent candidates but also a significant minority who should never have been entered for this examination. There was evidence in some responses that the whole syllabus had not been covered.

Question 1

This question was well answered in general although some of the weaker candidates differentiated the whole expression rather than the numerator and denominator separately. Some candidates wrote $\operatorname{cosec} x - \cot x = \frac{1}{\sin x} - \frac{1}{\tan x} = \frac{\tan x - \sin x}{\sin x \tan x}$ which is correct but it introduced an extra round of differentiation with opportunity for error.

Question 2

This question was well answered in general although a minority appeared not to realise that Diophantine means a solution in integers. It was expected that candidates would find a particular solution by inspection but some took a little longer by going via the Euclidean Algorithm.

Question 3

This question was well answered in general although some candidates showed only commutativity, not realising that they also had to prove that it was a group.

Question 4

This question was very well answered in general. In (d), some candidates stated that G is planar because $e \le 3v - 6$. It is however important to realise that this condition is necessary for a graph to be planar but not sufficient. Some candidates stated that G is planar 'because I have drawn it as a planar graph' or even 'see graph in (a)'. Candidates were expected to state that G is planar because it can be drawn with no edges crossing.

Question 5

Solutions to this question were often disappointing. Candidates were expected to use appropriate software on their calculators to do the whole question. However, some candidates used their calculators just to evaluate sums and sums of squares and then used the appropriate formulae to calculate the correlation coefficient, the *p*-value (which required the evaluation of the *t*-value first) and the equation of the regression line. This was a time consuming exercise and introduced the possibility of arithmetic error.

Question 6

This question was well answered in general although the presentation was sometimes poor.

Question 7

Part (a)(i) was well answered in general. In (a)(ii) and (b), however, many candidates made the fairly common error of confusing $\sum_{i=1}^{n} X_i$ with nX which gives an incorrect variance. This is an important distinction which needs to be emphasised.

Question 8

Many candidates made the connection between (a) and (b) and went on to solve the differential equation correctly. Candidates who failed to make the connection usually tried to write the equation in the form required for the use of an integrating factor but this led nowhere.

Question 9

This question was very well answered in general, although some candidates failed to see that $121 = 11^2$ in all number bases greater than 2.

Question 10

Many candidates were unable to find the coordinates of the point A which made (b) inaccessible. Many candidates reached the halfway point in (b) but were then unable to use the half angle formulae to obtain the required result. Many of the candidates who failed to solve (b) picked up the A1 in (c) for finding the gradient.

Question 11

Most candidates were able to show that *f* was an injection although some candidates appear to believe that it is sufficient to show that $f(x, y)$ is unique. A significant minority failed to show that *f* is a surjection and most candidates failed to note that it had to be checked that all values were integers. Some candidates introduced a matrix to define the transformation which was often a successful alternative method.

Question 12

Many solutions to this question suggested that the topic had not been adequately covered in many centres so that solutions were either good or virtually non existent. Most successful candidates used their calculator to perform the row reduction.

Question 13

Many candidates made no significant attempt at this question. It was expected that solutions would use the intersecting chords theorem but in the event, the majority of candidates who answered the question used similar triangles successfully to prove the required result.

Question 14

This proved to be a difficult question for most candidates with only a minority giving a correct solution. Most candidates either made no attempt at the question or just wrote several lines of irrelevant mathematics.

Question 15

Most candidates attempted this question with many showing correctly that ρ_1 is an equivalence relation. Most candidates, however, were unable to find a counterexample to show that ρ_2 is not transitive although many suspected that was the case. Most candidates were unable to describe the equivalence classes.

Question 16

This was the worst answered question on the paper and indeed no complete solution was seen with no candidate having the required insight to write down the solution. Most candidates who attempted the question tried to find the points of intersection of the line and circle which led nowhere.

Recommendations and guidance for the teaching of future candidates

Candidates should be made aware of the full range of the software on their calculators, particularly in statistics.

It was evident on some responses that linear algebra and geometry were only covered superficially in some centres.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 17	18 - 35	36 - 53	54 - 73	74 - 92	93 - 112	113 - 150

General comments

Overall the paper seemed to be well-received by candidates with many students showing good knowledge across all areas of the syllabus. However a small number of candidates had clearly only understood a limited amount of the syllabus or were relying on the knowledge they had gained from the HL option they had been taught.

The areas of the programme and examination which appeared difficult for the candidates

On this paper candidates found difficulty with transformations, the Lagrange form of the error term for Taylor series, and homomorphism of a group.

The areas of the programme and examination in which candidates appeared well prepared

On the whole candidates appeared to have been reasonably well prepared for questions on Euler's method, recurrence relations and matrices.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Most candidates were successful in applying Euler's method and in explaining how it could be improved to provide a better approximation. In part c) many candidates successfully used an integrating factor to solve the differential equation but a significant minority were unable to make a meaningful start. Part d) produced many fully correct answers, but candidates sometimes used their own answers to part c) to derive the Maclaurin series rather than the given equation. In most cases this did not cause a problem but a small number of candidates produced an expression of such complexity that they were unable to differentiate to the required number of terms.

Question 2

Almost all candidates recognised the sample distribution as normal but were not always successful in stating the mean and the standard deviation. Similarly almost all candidates knew how to find an unbiased estimator for μ, but a number failed to find the correct answer for the unbiased estimator for σ^2. Most candidates were successful in finding the 95% confidence interval for μ. In part c) many fully correct answers were seen but a significant number of candidates did not recognise they were working with a t-distribution.

Question 3

Students often gained full marks on parts a) and b), but a minority of candidates made no start to the question at all. In part c) it was pleasing to see a number of fully correct solutions to the strong induction, but many candidates lost marks for not being fully rigorous in the proof.

Question 4

Parts a) and b) were well done by most candidates, but surprisingly many candidates lost marks on part c). Parts e) and f) were only completed successfully by a small number of candidates and it was common to see parts a) and b) fully correct, parts c) and d) attempted but not fully correct and parts e) and f) not attempted at all.

Question 5

This proved to be a more challenging question for many candidates. In part a) many candidates appeared to not know how to find the matrices and for those who attempted to find them, arithmetic errors were common. A number of wholly correct solutions to parts b), c) and d) were seen, but many candidates seemed unfamiliar with this style of question and made errors or simply gave up part way through the process.

Question 6

It was pleasing to see many correct answers to parts a) and b) with candidates correctly recognising how to work with the distribution. Part c) caused more problems. Although a number of wholly correct solutions were seen, many candidates were unable to work meaningfully with the conditional probability.

Question 7

Part a) was successfully answered by the majority of candidates. There were some wholly correct answers seen to part b) but a number of candidates struggled with the need to formally explain what was required.

Question 8

Part a) was answered successfully by most candidates. However, the majority of candidates struggled to gain full marks on the remainder of the question. In part b) candidates struggled to work out which angle to use to find the maximum value. In part d) most candidates understood that this was related to a translation of the sine graph but were unable to explain it convincingly.

Question 9

It was pleasing to see a small number of wholly correct responses on this final question. Although the majority of candidates gained some marks, the majority failed to gain full marks because they failed to show full formal understanding of the situation.

Recommendations and guidance for the teaching of future candidates

It was pleasing to see a number of candidates make good and meaningful attempts at all questions, showing a good overall knowledge of the whole syllabus. However, many scripts were seen where candidates would answer questions on specific topics almost wholly correctly and then other questions would not be attempted at all, suggesting they were relying on the option they have covered for Mathematics HL with a small amount of information on the other topics. Unless all six topics are covered fully it is unlikely that candidates will be successful.

A number of students were let down by not appreciating the level of formality and precision needed in terms of what they write. Within a Further Mathematics HL course this is a requirement.