May 2012 subject reports

FURTHER MATHEMATICS

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-13	14-27	28-39	40-50	51-60	61-71	72-100

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-9	10-19	20-28	29-35	36-41	42-48	49-60

The areas of the programme and examination that appeared difficult for the candidates

Candidates continue to find questions on pure geometry difficult.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are extremely competent in using the calculator to solve problems involving statistical inference.

Most students are comfortable with questions involving specific finite groups although more theoretical questions on groups are not so well answered.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Parts (a) (i) and (a) (iii) were well answered in general. However, in (a) (ii), some candidates lost marks by not showing convincingly that $\{S_1, \times_{10}\}$ was a group. For example, in verifying the group axioms, some candidates just made bald statements such as '$\{S_1, \times_{10}\}$ is closed'.

This was not convincing because the question indicated that it was a group so that closure was implied by the question. It was necessary here to make some reference to the Cayley table which showed that no new elements were formed by the binary operation. To gain full marks on this style of question candidates need to clearly explain the reasoning used for deductions. In (b), most candidates realised that the quickest way to establish isomorphism (or not) was to determine the order of each element. Candidates who knew that there are essentially only two different groups of order four had a slight advantage in this question.

Question 2

As expected, the factorisation in (a) was successfully completed by most candidates. Part (b) caused problems for some candidates. The most common mistake was that only one pair of values for *M*, *N* was given.

Question 3

A reasonable number of candidates achieved full marks on this question. However, in part (a) a number of candidates struggled to find the Maclaurin series accurately. It was not uncommon to see errors in finding the higher derivatives, which was often caused by not simplifying the answer for earlier derivatives. At this level, it is expected that candidates understand the importance of using the most efficient methods. A pleasing number of candidates made significant progress or achieved full marks in part (b), provided that they realised the importance of recognising that

$$\frac{\ln \cos x}{x^n} = -\frac{x^{2-n}}{2} - \frac{x^{4-n}}{12} + \ldots$$

Question 4

Questions on pure geometry which require an initial construction to be made are usually either well done or not done at all and this was no exception. The obvious construction was to draw the line BD and use Menelaus' Theorem twice although some candidates took the more difficult route by joining AC which also leads to the solution. However, many candidates did not start the question at all or tried to apply Menelaus' Theorem to existing triangles which was not a successful approach. The examiners saw a number of candidates who produced well set out and well-explained solutions, but there were still a significant number of cases where diagrams were not fully labelled and points were referred to in the working that were not on the diagram. Candidates should realise that to ensure full marks on questions involving geometric proof that there is a certain degree of formality required in the solution.

Question 5

Part (a) was well answered in general, using the calculator either to carry out a significance test on proportions or to find the *p*-value directly using the binomial distribution. Some candidates gave their conclusion in the form 'Accept H_0'; this was not accepted since the question asked whether or not the shopkeeper's claim was supported and a direct answer to this question was required. Part (b) caused problems for some candidates who were unsure how to proceed. Some candidates used a trial and error method which involved showing that

$\hat{p} = 0.4$ and then using their calculator to find the confidence interval for appropriate pairs of values for *n* and *p* until reaching 400,160. This was accepted as a valid method although it is not recommended as a general method since its success was based upon the value of *n* being one that would probably be tested.

Question 6

A significant number of candidates struggled with this question, but a number of wholly correct answers were seen. The majority of candidates tried to use a method of proof by induction although a number got lost in trying to establish the result for *k*+1. Again the presentation and explanation of some of the solutions was poor. Some excellent solutions were seen using other methods including one which noted simply that all positive integers *n* are either 0 or ±1 modulo 3 so that $n(n^2+2) \equiv 0$ modulo 3 for all *n*.

Recommendations and guidance for the teaching of future candidates

As in previous years there are indications that a number of candidates had not studied the whole syllabus. Unless candidates do study the whole syllabus, they will not be able to achieve their full potential.

In questions involving proof, there is often a lack of formality, clarity and precision in what students write. Examiners are fully aware that candidates are under a pressure of time when doing this paper but candidates should realise that formality, clarity and precision are a part of proof and hence needs to be demonstrated even in an examination situation.

The timing allowed for this examination assumes that good candidates will understand and be able to use efficient methods for solving questions. Not simplifying as candidates work through a questions is likely to result in errors and in having difficulties with timing.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-15	16-30	31-42	43-54	55-67	68-79	80-120

The areas of the programme and examination that appeared difficult for the candidates

The question on bijections in 2-D caused problems for many candidates.

Many candidates appeared to be unfamiliar with cumulative distribution functions.

The areas of the programme and examination in which candidates appeared well prepared

In general, candidates appear to be well able to handle specific problems in graph theory and modular arithmetic.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1 – Part A

Most candidates attempted (a) with many different methods seen although some candidates made algebraic errors in applying the appropriate trigonometrical formulae. Very few candidates realised that, because the centre of the triangle divides an altitude in the ratio 2:1, the area of the inscribed circle is one quarter the area of the circumscribed circle.

Question 1 – Part B

Most candidates solved (a) correctly but solutions to (b) were often disappointing. Those candidates who used coordinate geometry often made algebraic errors in obtaining the equation of the circle and then finding the coordinates of its centre. Some candidates tried to use Apollonius' theorem, using the fact that the centre divides the line AB in a ratio dependent upon *k,* but this approach almost invariably led to algebraic errors.

Question 2 – Part A

Solutions to (a) and (b) were generally satisfactory. In (c)(ii), few candidates realised that they had to find the number of walks of length two joining A to F, the number of walks of length three joining F to B and then multiply these two numbers together. In (d), most candidates noted that the number of edges, *e*, was equal to 8 and that application of the inequality $e \le 2v - 4$ gave $e \le 10$. They therefore concluded that two more edges could be drawn. It is, however, important to realise that the value of *e* given by this inequality is an upper bound that may not be attainable and that in this case, it was necessary to show that two extra edges could in fact be drawn.

Question 2 – Part B

This question was well answered in general with a variety of methods seen. Most candidates realised that the numbers involved precluded the use of Fermat's little theorem. In (c), most candidates gave $x = 3$ as a solution following their earlier work in (a) but many candidates failed to realise that their answer to (b) showed that the general solution to (c) was actually 3 + 5*N* where *N* is a non negative integer.

Question 3

Most candidates managed to solve (a) successfully although some solutions required a page or more to complete with candidates rewriting $\sec\theta$ and $\tan\theta$ in terms of $\sin\theta$ and $\cos\theta$ which increased the complexity of the problem and sometimes led to algebraic errors. Most candidates made a good attempt at (b) (i), those candidates who gave their solution in tabular form being most successful. In (b)(ii), most candidates found the correct integrating factor but many were unable to solve the differential equation in (b)(iii) with some failing to see that the result in (a) was intended as a hint for an appropriate substitution.

Question 4 – Part A

This proved to be a difficult question for some candidates. Most candidates realised that they had to show that the function was both injective and surjective but many failed to give convincing proofs. Some candidates stated, incorrectly, that *f* was injective because ***AX*** is uniquely defined, not realising that they had to show that $\boldsymbol{AX} = \boldsymbol{AY} \Rightarrow \boldsymbol{X} = \boldsymbol{Y}$. Solutions to (b) were disappointing with many candidates failing to realise that they had either to show that ***AX*** was confined to a subset of R×R or that two distinct vectors had the same image under *f*.

Question 4 – Part B

This question was well answered in general with solutions to (c) being the least successful.

Question 5

Solutions to (a) were often unconvincing. Candidates were expected to include in their solution the fact that $F(a) = 0$ and $F(b) = 1$. In (b) (i), it was not enough to state that $G(y) = \int \cos y \, \mathrm{d}y = \sin y$ although that, fortuitously, gave the correct answer on this occasion. The correct approach was either to state that $G(y) = \int_0^y \cos t \, \mathrm{d}t = \sin y$ or that $G(y) = \int \cos y \, \mathrm{d}y = \sin x + C$ and then show that $C = 0$ because $F(0) = 0$ or $F\frac{\pi}{2}) = 1$. Solutions to (b) (ii) and (iii) were often disappointing, giving the impression that many of the candidates were not familiar with dealing with cumulative distribution functions.

Recommendations and guidance for the teaching of future candidates

Experience shows that, in general, candidates are confident in using the calculator to solve inferential problems in statistics. The more theoretical problems in statistics, however, are not so well answered and this, perhaps, is an area that should be emphasized.

Many candidates were unable to solve the question on bijections in 2-D and this is clearly an important situation to consider.