FURTHER MATHS

Overall grade boundaries

Standard level

Grade:	1	2	3	4	5	6	7
Mark range:	0-18	19-36	37-48	49-58	59-69	70-79	80-100

Standard level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-11	12-22	23-29	30-35	36-42	43-48	49-60

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Candidates used a variety of methods to find a limit. Nearly all found the correct limit. In part (b) some candidates ignored the modulus sign, added ½ to 0.001 and then considered $|a_n| < 0.501$. Some paid no attention to the inequality, solved an equation and then rounded the answer obtained up or down without any reasoning. Many, however, successfully found the correct value for N. Alarmingly though there were candidates who gave a negative decimal solution for N and this demonstrated a total lack of understanding of the concept of the limit of a sequence.

Question 2

This question presented the greatest challenge for the candidates. Nevertheless, it is a relatively straightforward proof which can be done using tangent theorems and congruent triangles. Most candidates were unable to make any significant statements resulting from the given information. Some even made such fundamental errors as stating that OLTM is a cyclic quadrilateral. Very few marks were earned on this question.

Question 3

Part (a) was correctly answered by nearly every candidate. There were many good proofs of part (b). Unfortunately, there were too many candidates who simply rambled on about even and odd numbers without making precise linking statements that came together to constitute a mathematical argument.

Question 4

Most candidates answered this correctly. Those who did not were those who failed to recognise that the function mapped positive integers to positive integers. They stated the range as a continuous interval from 1 to 6 inclusive. A common error on part (b) was in the method laid out by the candidates to "show that" the function is periodic. Whereas a tabular approach proved the simplest, some candidates only evaluated 6 values of $f(x)$, which was insufficient to demonstrate the periodicity of f. Some drew a graph of f for $1 \le x \le 12$ but again it was surprising to see that some drew a continuous graph. In part (c) candidates generally recognised that $f(2) = f(4) = 2$. Then a set of values {2, 4, 8, 10,…} was produced but without general terms to represent this set. For those who did attempt to give the general terms there was often an error in the domain. For instance, many incorrectly wrote: $\{2+6k, 4+6k, k \in \mathbf{Z}^+\}$. Others expressed the general solution in terms of multiples of 2 excluding multiples of 6, which was acceptable.

Question 5

Nearly all candidates omitted to state that $f(x) \geq 0$ for $x \geq 0$. Others tried to just state that this is an exponential probability density function of a specified form. They failed to recognise that this is what they were required to show. The majority of candidates, nonetheless, correctly integrated the improper integral from 0 to infinity to get a result of 1. In part (b) there were some candidates who did not know how to find the expected frequencies. Others correctly computed the first 3 values then, for the final value, instead of finding the difference between the sum of these three values and 150, they integrated a definite integral with an upper limit that was too small to give the correct expected frequency. Some other candidates rounded the expected frequencies to two significant figures when it would have been better to carry four figures. Many successfully performed a chi squared goodness of fit test, either by using a comparison of the calculated chi squared value to the critical value, or by using p values. The conclusion, for the most part, was correct although some candidates failed to state the null and alternate hypothesis.

Question 6

Whereas many candidates knew how to use a substitution $y = vx$ to solve this homogeneous differential equation, the majority of them could not show that it is homogeneous. Since the instruction was to "show that", it was not sufficient to define the required form of this type of differential equation and then just state: "this equation is of that form". The steps to transform the equation to that form were required. Most of those candidates who knew how to solve by substitution were able to complete the question and find the required particular solution. There were candidates, though, who showed a complete lack of understanding of what was required. Some of these tired to use an integrating factor. Others, surprising at what is supposed to be the highest level of mathematics, showed no understanding of the idea of separation of variables, or even of basic integration concepts. Some treated variables as constants and wrote $\int xy \, \mathrm{d}y = \frac{xy^2}{2}$ or $\int \left(3x + y^2\right) \mathrm{d}x = \frac{3}{2}x^2 + y^2 + C$ or $\int y^2 \frac{\mathrm{d}x}{x} = y^2 \int \frac{\mathrm{d}x}{x}$. There were candidates who were unable to find the general solution and had the mistaken idea that they could just make up a general solution, use the initial values and then get marks for finding a particular solution. Finally some candidates did not apparently understand what is meant by "solve the differential equation" and after finding the correct particular solution they set the $y-$value equal to zero and found the corresponding values of x.

Standard level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-21	22-42	43-58	59-70	71-82	83-94	95-120

The areas of the programme and examination that appeared difficult for the candidates

The areas which caused most problems in general were certain aspects of probability and statistics and geometry.

The areas of the programme and examination in which candidates appeared well prepared

The overall standard of the candidature was somewhat variable with some very good candidates but also some who were clearly unprepared for an examination at this level. In general, the topics that are best understood are sets, relations and groups, series and differential equations and discrete mathematics.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Part A

In (a), most candidates answered (i) correctly but (ii) caused problems. Some solutions gave the impression that some candidates had not been introduced to the negative binomial distribution.

In (b), many candidates found the mean correctly but the standard deviation proved too difficult for some, in spite of the appropriate formula being in the Information Booklet.

Part B

In (b), some candidates seemed unfamiliar with the term 'critical region'.

In (d), many candidates stated, incorrectly, that a Type I error was being committed.

Question 2

Part A

In (b), to prove closure, some candidates showed that

$$\begin{bmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a & b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2a & 2b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

It is important to realise that this particular result is not sufficiently general to prove closure.

In (c), many candidates provided a semi-rigorous justification of isomorphism but few gave what could be regarded as a fully acceptable proof.

Part B

Most candidates solved (a) correctly although some took a page of repetitive algebra to do it.

In (b), details of the proof were sometimes incorrect, eg in establishing symmetry some candidates wrote $xRy \Rightarrow x = z \bullet y \bullet z^{-1} \Rightarrow y = z^{-1} \bullet x \bullet z \Rightarrow yRx$ without realising that it was necessary to show that $y = z^{-1} \bullet x \bullet (z^{-1})^{-1}$.

Question 3

Part A

Part (b) was well answered by many candidates, with most using the method given in the standard textbook.

Part B

The intention of Part (a) was to test the planarity condition $e \le 3v - 6$. Many candidates simply stated that κ_5 is non-planar and is a subgraph of κ_6 whence the result. This was given only partial credit on the grounds that assuming that κ_5 is non-planar is hardly different from assuming that κ_6 is non-planar which was the result that candidates were asked to prove.

In (b), some candidates started from a vertex other than B, for which partial credit was given.

Question 4

Part A

Several candidates solved this question by solving the equation

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \sin x = 0.005$$

which gives $x = 1.59$ and then obtaining the correct result. Even though the question was intended to be an exercise in the use of remainders, this was accepted as an alternative solution.

Part B

This was well answered by many candidates with the correct use of several different tests seen.

Question 5

This was well answered by many candidates although some failed to write down all the working which was penalised in view of the new marking instructions.

Question 6

Solutions to (a) were often disappointing. Most candidates realised that the quadrilateral AFBE is cyclic but some were unable to use Ptolemy's Theorem correctly.

In (b), many candidates realised that Menelaus' Theorem was needed but not all were able to apply it correctly.

Recommendations and guidance for the teaching of future candidates

- Ensure that the candidates are fully aware of the rules on the accuracy of answers so that accuracy penalties are not given.
- Ensure that candidates are aware that marks can be lost if not all relevant working is shown.
- Ensure that candidates are familiar with all the probability distributions in the new syllabus,
- Concentrate a little more on geometry.

